

AN ANALYSIS OF A CLASS OF LANCHESTER-TYPE
WARFARE MODELS

A THESIS


Presented to
The Faculty of the Division of Graduate Studies
By
Dennis Milton White

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in the School of Industrial
and Systems Engineering

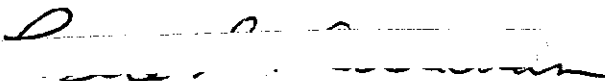
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WARFARE MODELS

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CHAPTER I

INTRODUCTION

Problem

There has been a recent resurgence of interest in Lanchester-type warfare models by military analysts. In addition, there is a growing interest at Georgia Tech. in the use of analog or syndetic analog-digital computer methods for studying parametrically by solving the dynamics of combat analytically intractable Lanchester-type models. It was hoped that a hybrid computer would be installed at Georgia Tech. within the time span of this thesis work, in order to begin a preliminary investigation as to the feasibility of analog-digital simulation of these types of models.

Objective

The objective of this thesis is to create a general digital model of Lanchester-type equations of infantry combat engagements that includes:

1. Time delays for artillery fire support.
2. Ammunition constraints on fire effectiveness.
3. Resupply of ammunition with associated delays.
4. Comprehensive set of measures of effectiveness (MOE).

This model can be used to experiment with, to show the sensitivity of various MOEs to changes in some of 40 model parameters, and to illustrate the magnitude of the computational tasks and the potential for syndetic calculations, coupled with convenient displays of results. This thesis can be a base for further investigation into Lanchester-type warfare models.

Beyond reviewing the existing literature, in itself an important task, this thesis will contribute to the structure of Lanchester-type models by including ammunition inventory constraints. It is clear that the dynamics of battle do depend on ammunition supply and constraints, and the model presented here is the first to consider these factors.

CHAPTER II

LANCHESTER EQUATIONS

Original Development

The first mathematical analysis of the relationship between opposing forces in combat was made by F. W. Lanchester (24) in 1916, and is known as Lanchester Equations of Warfare. These models are simultaneous differential equations which describe the loss rates of the opposing sides over time. Lanchester's original work was formulated to depict combat between two homogeneous forces in order to justify the principle of concentration of forces under modern conditions of "aimed fire". Two laws were formulated by Lanchester, the Linear Law and the Square Law.

Linear Law

Lanchester's linear law applies to either individual duels between members of opposing forces or combat situations where firepower is randomly directed on an area known to be occupied by the opposing force.

In the case of individual duels, the differential models are:

$$\frac{dB}{dt} = - KN \quad (1)$$

$$\frac{dR}{dt} = - JN$$

where

$\frac{dB}{dt}$, $\frac{dR}{dt}$ = the attrition rate of the BLUE and RED forces, respectively, with respect to time,
 J , K = the coefficient of effectiveness of the BLUE and RED forces, respectively, and
 N = the number of duels per unit of time.

In the case of randomly directed firepower, the differential models are:

$$\frac{dB}{dt} = - KB(t)R(t) \quad (2)$$

$$\frac{dR}{dt} = - JB(t)R(t)$$

where $B(t)$ and $R(t)$ are the number of surviving units of BLUE and RED forces, respectively, at time t .

Equations (1) is a special case of Equations (2) where $B(t) = R(t) = 1$ in an individual duel and Equations (2) becomes:

$$\frac{dB}{dt} = -K$$

$$\frac{dR}{dt} = -J$$

When the number of duels per unit of time are considered, Equations (1) are obtained.

The solution to Equations (2) is:

$$\begin{aligned} B(t) &= \frac{B_0(KR_0 - JB_0)}{KR_0 e^{(KR_0 - JB_0)t} - JB_0} \\ R(t) &= \frac{R_0(JB_0 - KR_0)}{JB_0 e^{(JB_0 - KR_0)t} - KR_0} \end{aligned} \quad (2')$$

where

B_0 = initial size of BLUE FORCE.

R_0 = initial size of RED force.

The solution to Equations (2) with time eliminated is:

$$J(B_0 - B) = K(R_0 - R) \quad (2'')$$

The above solution (2'') is also a solution to Equations (1), and indicates that there is no particular advantage in concentrating forces. That is, the effect of force size is linear.

Lanchester (24) stated that two sides are equally matched when the ratio of their attrition is equal to the ratio of their numerical strengths, or

$$\frac{dB}{dR} = \frac{B}{R}$$

By substituting the values for dB and dR from Equations (1) into the above equality results in:

$$\frac{K}{J} = \frac{B}{R}$$

This implies that for the linear law, two opposing forces are equally matched when the ratio of their numerical strength is equal to the reciprocal of the ratio of their effectiveness.

Square Law

Lanchester's square law applies to the case where the opposing forces are exposed to each other and execute aimed fire at a given rate with given single-shot hit probability. In this case it is assumed that each participant can fire at any member of the opposing force.

The differential models for this case are:

$$\frac{dB}{dt} = -KR(t) \quad (3)$$

$$\frac{dR}{dt} = -JB(t)$$

The solution to Equations (3) is:

$$B(t) = B_0 \cosh t \sqrt{JK} - R_0 \sqrt{K/J} \sinh t \sqrt{JK} \quad (3')$$

$$R(t) = R_0 \cosh t \sqrt{JK} - B_0 \sqrt{J/K} \sinh t \sqrt{JK}$$

The solution to Equations (3) with time eliminated is:

$$J(B_0^2 - B^2) = K(R_0^2 - R^2) \quad (3'')$$

The condition for equality of fighting strength is seen to be:

$$\frac{K}{J} = \frac{B_0^2}{R^2}$$

This implies that two opposing forces are equally matched when the square of the ratio of their numerical strengths is equal to the reciprocal of the ratio of their effectiveness. Thus, for the square law, it is more advantageous to increase the number of participants in a battle than it is to increase by the same factor the effectiveness of weapons.

Shortcomings of Original Models

Although Lanchester certainly sparked the interest in the mathematical modeling of combat, there were many shortcomings to his original models. Taylor (39) sums up these shortcomings as:

1. Coefficients constant over time.
2. No movement of forces (e.g. advance or retreat).
3. Considers homogeneous forces only.
4. Battle termination conditions not given.
5. Deterministic, not probabilistic.
6. No replacements or withdrawals.
7. Target acquisition force level independent.
8. Fire allocation not explicitly considered.
9. No consideration of noncombat losses (e.g. surrenders, desertions).
10. No logistic considerations.
11. No way of predicting Lanchester attrition-rate coefficients.
12. Suppressive effects of weapons not considered.
13. Effects of terrain not considered.

Extension of Lanchester Theory

Since Lanchester's original work in 1914, military analysts have employed simple deterministic differential equation models to obtain insights into the dynamics of combat, even though combat between two military forces

is a complex random process. World War II provided a major stimulus for military scientists to expand, test, and use the primordial Lanchester models employing analytical solution methodologies and using field data.

Many of the shortcomings in Lanchester's original models have been investigated. For example, the idea of time varying attrition-rate coefficients has been investigated by Barfoot (2), Bonder (3, 4), Karr (20), Snow (32), Taylor (33, 36), and others by estimating the coefficients from weapon system performance data. Clark (10) developed attrition-rate coefficient estimates using statistical (maximum likelihood) estimation from Monte Carlo simulation output.

Work has been done by Helmbold (18) and Weiss (42, 43) concerning battle termination conditions. Their conclusions were that the force levels alone are not the significant variables for predicting battle outcome as one might be led to believe.

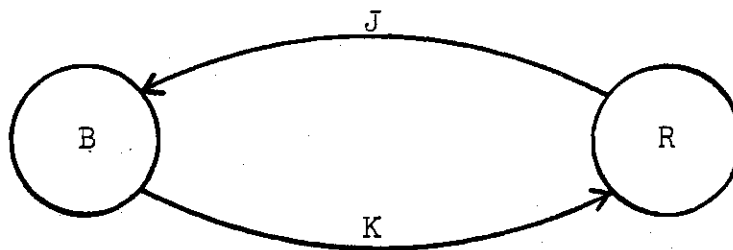
In the area of the probabilistic nature of combat between two forces, rather than the deterministic case, as was in Lanchester's original work, many authors (4, 6, 7, 8, 10, 20, 25, 31, 41) have added contributions to this field. The general conclusions, as depicted by Taylor (39), are that biases exist between the deterministic and stochastic models, with the stochastic model being more realistic.

Figure 1 shows the difference in model formulation between the deterministic and stochastic models. Figure 2 shows the bias that results between the two models.

Kisi and Hirose (22) and Schaffer (29) have done much to contribute to the study of Lanchester equations depicting ambush engagements. Schaffer includes in his model, the morale and discipline coefficients associated with surrenders and desertions (noncombat losses) from both the combat forces and the reserve replacement forces.

Recent Developments

The development of large capacity digital computer technology in the 50's and 60's made possible the modeling and simulating of large numbers of physical and tactical variables, and resulted in a variety of specialized stochastic simulations. Relatively little attention was directed at improving Lanchester models. By the early 1970's, there was increased concern as to the efficiency and cost of these large-scale, stochastic simulations. Lanchester-type models were thought of as suggesting a useful means of aggregating the myriad of combat environmental variables into attrition-rate coefficients and force levels. In addition to the adaption of the Lanchester-type model to complementing the present generation of large-scale simulations, there appears to be an increased interest in extending them to analyze



DETERMINISTIC MODEL	STOCHASTIC MODEL (Average Force Level)
$\frac{dB}{dt} = -JR$	$\frac{d\bar{m}}{dt} = -J\bar{n} + J \left\{ \sum_{n=0}^{n_0} nP(t, o, n) \right\}$
$\frac{dR}{dt} = -KB$	$\frac{d\bar{n}}{dt} = -K\bar{m} + K \left\{ \sum_{m=0}^{m_0} mP(t, m, o) \right\}$
<p>where</p> <p>$B(t), R(t)$ are force levels</p>	<p>where</p> <p>$\bar{m}(t), \bar{n}(t)$ are average force levels</p>
<p>Initial values:</p> <p>$B(t=0)=B_0, R(t=0)=R_0$</p>	<p>Initial values:</p> <p>$\bar{m}(t=0)=m_0, \bar{n}(t=0)=n_0$</p>

Figure 1: Deterministic vs Stochastic Models

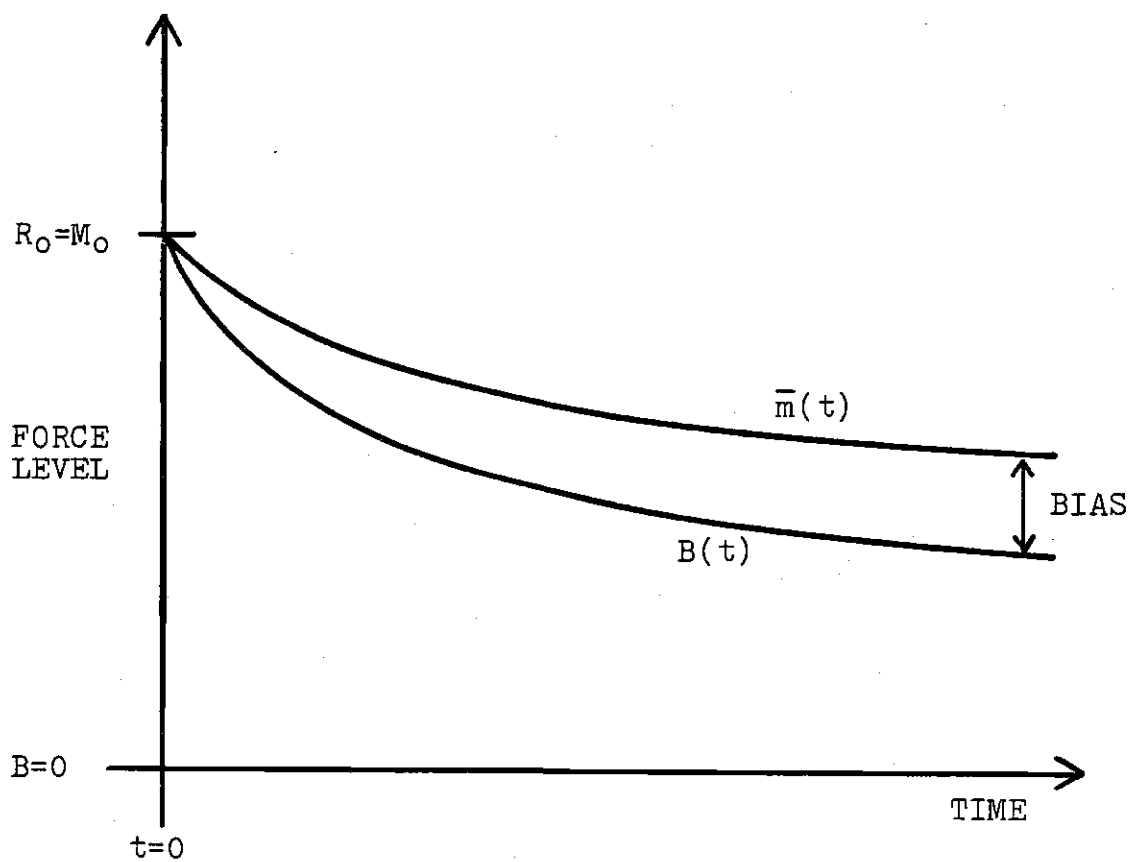


Figure 2: Comparison of Results from Deterministic and Stochastic Attrition Models

problems of replenishment, fire direction and tactical situations involving troop movement and both direct and indirect firing.

Theoretical work by Schaffer, Taylor, Weiss, and others (1, 5, 9, 14, 19, 21, 23, 29, 33, 36, 44) has again extended the classical Lanchester formulation by including simple attrition factors (e.g. desertion or withdrawal), the effect of suppressing fire, battle termination rules, and time-varying parameters with simple form. These models have been constrained to allow closed-form analytical solution. Current work by Willis (45), for example, has begun to deal with more complex models which trade analytical tractability for increased realism. Sources of complexity are highly non-linear equations for force attrition, non-linear output measures, and the $n \times n$ problem (combat involving more than two homogeneous forces).

The current mathematical interests in Lanchester-type equations are trying to predict battle outcomes from initial time rates, developing generalized cosh/sinh functions for the solution to time-varying coefficients, and the area of differential games.

For the reader's future reference, a selected bibliography on Lanchester-type models of warfare is included. Most of these references are journal articles that have appeared in the open literature, but represent a fairly complete list in the area.

CHAPTER III

GENERAL LANCHESTER MODEL

Model Equation Development

There have been many different Lanchester-type models developed for the analysis of combat engagements, with rarely any two alike. After reviewing the literature on existing models, it was decided to start by using Lanchester's original model for the square law as in Equations (3). To this model was added:

1. Losses due to surrenders and desertions.
2. Rate of fire for infantry weapons.
3. Suppression factors due to opposing artillery fire.
4. A gamma function to allow for switching from aimed fire to area fire.
5. Ammunition constraint functions on fire effectiveness.
6. Supporting artillery fire.

A general causal diagram depicting this model is shown in Figure 3. After taking Lanchester's original model and adding the above listed functions, the following set of equations were constructed to represent this model:

$$\frac{dB}{dt} = -K(1-S_B)B(t)^{\gamma} r_R f(A_R) - D_B - \alpha_R W_R B(t) \quad (4)$$

$$\frac{dR}{dt} = -J(1-S_R)R(t)^{\gamma} r_B f(A_B) - D_R - \alpha_B W_B R(t)$$

where

- J, K = the coefficient of effectiveness of the BLUE and RED forces, respectively,
- S_B, S_R = suppression factor for BLUE and RED forces due to opposing fire,
- $B(t), R(t)$ = number of surviving BLUE and RED forces at time t ,
- $B(t)^{\gamma}, R(t)^{\gamma}$ = form of the law, where $\gamma=0$ is the square law and $\gamma=1$ is the linear law,
- r_B, r_R = rate of fire (rounds per minute) for BLUE and RED forces,
- $f(A_B), f(A_R)$ = ammunition constraint functions for BLUE and RED forces,
- D_B, D_R = losses due to surrenders and desertions for the BLUE and RED forces,
- α_B, α_R = coefficients of effectiveness for the supporting artillery fire for the BLUE and RED forces, and
- W_B, W_R = number of supporting artillery weapons for the BLUE and RED forces.

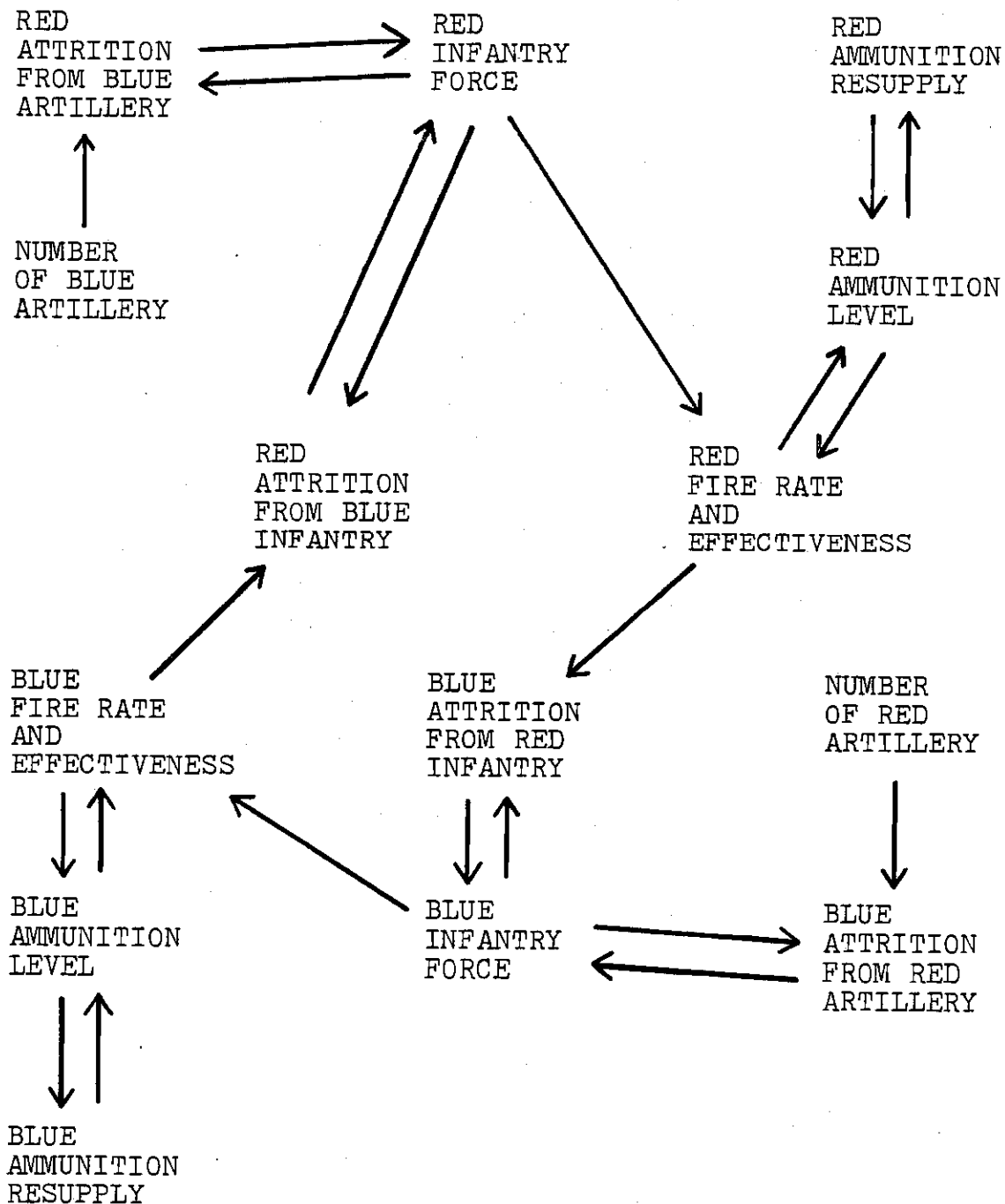


Figure 3. Causal Diagram: Lanchester Model

There is no general solution available for the above model; however, accurate solutions can be obtained through the use of high speed digital simulation. A computer program for the solution and analysis of this model was written in FORTRAN, and a complete listing of the model is in Appendix C.

This model has 40 parameters that can be varied by the user at the start of each run. For a series of battle engagements, the parameters can be made either permanent or temporary. A permanent parameter change remains in effect for the entire series of runs, whereas a temporary parameter change, applies only to the current run. A history of the 40 parameters is shown in Figure 4. The parameters and their values will be discussed later in this chapter, as will the different parts of the model equations, with the exception of the losses due to surrenders and desertions, which will be discussed in Chapter IV.

Ammunition Constraint Functions

Before military engagement, prior planning must be conducted in ammunition logistics to include both the amount of ammunition to be carried per soldier, and the resupply of ammunition if necessary. The amount of ammunition carried per soldier is called the "basic load" (parameters 21 and 22), and is normally standard for each soldier for a particular type of weapon. The weight of the

- 1 SOLUTION STEP SIZE
- 2 PRINT PERIOD
- 3 PLOT PERIOD
- 4 VARIABLE STEP SIZE MAX %
- 5 END BATTLE FORCE RATIO
- 6 INITIAL BLUE INFANTRY
- 7 INITIAL RED INFANTRY
- 8 GAMMA DENSITY CORRECTION
- 9 RED AMMO CRIT MIN INV
- 10 BLUE AMMO CRIT MIN INV
- 11 BASE EFF BLUE INF FIRE
- 12 BASE EFF RED INF FIRE
- 13 BASE EFF BLUE ARTY FIRE
- 14 BASE EFF RED ARTY FIRE
- 15 BASE RATE FIRE RED INF
- 16 BASE RATE FIRE BLUE INF
- 17 RED S+D PER RED CASUAL
- 18 BLUE S+D PER BLUE CASUAL
- 19 RED S+D DUE FORCE RATIO
- 20 BLUE S+D DUE FORCE RATIO
- 21 INT BLUE AMMO FER MAN
- 22 INT RED AMMO PER MAN
- 23 DELAY CALL BLUE ARTY
- 24 DELAY CALL RED ARTY
- 25 % RED SUPP OF BLUE INF
- 26 % BLUE SUPP OF RED INF
- 27 NUM BLUE ARTY TUBES
- 28 NUM RED ARTY TUBES
- 29 BLUE INF TV EFF AT ZERO
- 30 BLUE INF TV EFF TC
- 31 RED INF TV EFF AT ZERO
- 32 RED INF TV EFF TC
- 33 BLUE AMMO CONS FIRE EFF
- 34 RED AMMO CONS FIRE EFF
- 35 BLUE RORD FT AMMO/MAN
- 36 RED RORD FT AMMO/MAN
- 37 BLUE ORD UP TO AMMO/MAN
- 38 RED ORD UP TO AMMO/MAN
- 39 BLUE AMMO DELAY TIME
- 40 RED AMMO DELAY TIME

Figure 4. Parameters for Lanchester Model

ammunition is the biggest factor in determining the basic load.

This model incorporates two functions associated with the ammunition constraints. First is the number of rounds fired per minute per soldier (parameters 15 and 16). If there is plenty of ammunition, each soldier would probably fire a fairly constant number of rounds per minute for the duration of the engagement. However, at some point in the engagement, as the ammunition supply gets lower, the soldier will start firing at a slower rate per minute in fear of depleting his ammunition supply completely. This model considers this point in terms of a critical ratio in minutes of ammunition (parameters 9 and 10) remaining per soldier. This "critical ratio" can be varied as a parameter in the model, and is approximated by the exponential distribution in the form of:

$$A_B, A_R = 1 - e^{-\frac{d}{e}}$$

where

d = the ratio of remaining ammunition per man,

e = the critical ratio of ammunition per man.

Figure 5 shows a graph of this function.

The second function associated with ammunition constraints is the function for target selection and fire efficiency. As the ammunition supply reaches the critical

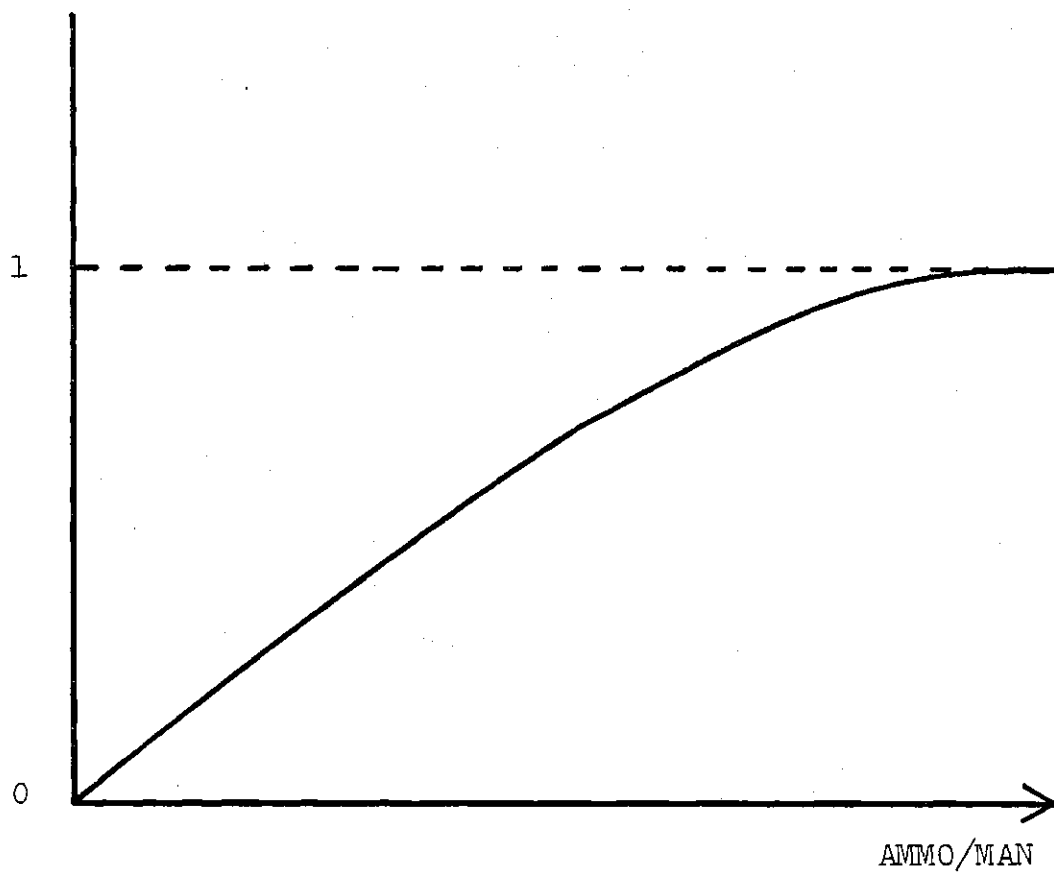


Figure 5. Function For Rounds Fired Per Soldier Per Minute

ratio, the soldier will not only fire fewer rounds per minute; but he will also be more selective in his target, and therefore, the probability of a hit per round fired increases. This function is also approximated by the exponential distribution in the form of:

$$1 - (1 - X)e^{-\frac{d}{e}}$$

where

X = maximum efficiency of firepower with d and e
the same as defined in the previous function.

A graph of this function is shown in Figure 6.

Ammunition Resupply

In the ammunition resupply sector, there are six parameters that can be specified for each engagement. First, it must be determined at what point in the engagement should the ammunition be ordered. This is specified in terms of the amount of ammunition remaining per man (parameters 35 and 36). Once the ammunition reorder point is reached, the amount of ammunition to order must be determined (parameters 37 and 38). There is then a delay time associated with the receipt of the ammunition after it has been ordered (parameters 39 and 40).

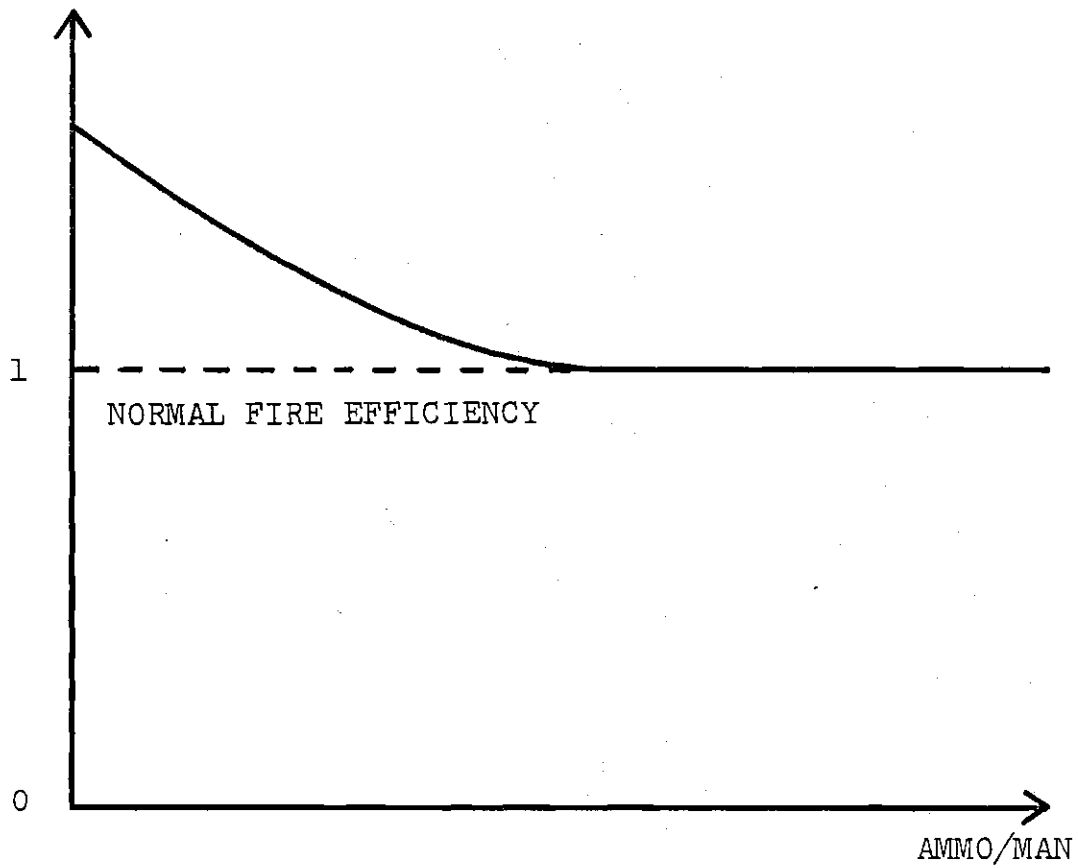


Figure 6. Function For Improved Fire Efficiency and Target Selection

Artillery Fire Support

The engagement using this model can be accomplished with or without artillery fire support. If there is going to be artillery fire support for the BLUE or RED or both, it is specified as parameters in the model. This is in terms of the number of artillery tubes available for each of the forces (parameters 27 and 28), and the base effectiveness of the firepower (parameters 13 and 14). Once an engagement starts, there is a delay in the time the artillery support is requested and the time it becomes effective (parameters 23 and 24). This delay time can also be varied by the user at the start of each engagement.

Battle Termination Rules

For this model, there are three conditions which will terminate the battle, when any of the conditions are met. First, the battle will terminate if the force ratio reaches a predetermined value. This value is one of the parameters and can be specified at the start of each run. The computation of this battle termination rule is as follows:

$$\frac{B(t)}{B_0(t)} < \beta$$

or

$$\frac{R(t)}{R_o(t)} < \beta$$

where

β = ending force ratio specified as a parameter.

The second battle termination rule is met if one of the forces has only one man left. The third battle termination rule is based on the amount of ammunition remaining per man. If the ammunition is depleted to only one round per man, the battle is terminated.

Measures of Effectiveness

To assist in analyzing the results of the battle engagements, 10 different Measures of Effectiveness (MOE) were established. Each of the MOE's are computed continually throughout the length of the battle. The following is a list and explanation of each of the MOE's:

1. MOE (1) is the number of BLUE infantry lost and is calculated by:

$$B_o(t) - B(t)$$

where

$B_o(t)$ = starting BLUE force size at time $t=0$, and

$B(t)$ = surviving BLUE force size at time t .

2. MOE (2) is the number of RED infantry lost and is calculated by:

$$R_o(t) - R(t)$$

where

$R_o(t)$ = starting RED force size at time $t=0$, and

$R(t)$ = surviving RED force size at time t .

3. MOE (3) is the present BLUE infantry lost and is calculated by:

$$(100) \left(\frac{B_o(t) - B(t)}{B_o(t)} \right)$$

4. MOE (4) is the percent RED infantry lost and is calculated by:

$$(100) \left(\frac{R_o(t) - R(t)}{R_o(t)} \right)$$

5. MOE (5) is the difference in percent lost between the BLUE and RED forces and is calculated by:

$$(100) \left(\frac{B_o(t) - B(t)}{B_o(t)} \right) - (100) \left(\frac{R_o(t) - R(t)}{R_o(t)} \right)$$

6. MOE (6) is the ratio of the number of BLUE to RED forces lost and is calculated by:

$$\frac{B_o(t)-B(t)}{R_o(t)-R(t)}$$

7. MOE (7) is the ratio of the percent of the original forces lost for the BLUE to RED and is calculated by:

$$\frac{(100) \left(\frac{B_o(t)-B(t)}{B_o(t)} \right)}{(100) \left(\frac{R_o(t)-R(t)}{R_o(t)} \right)}$$

8. MOE (8) is the difference in the number of BLUE and RED forces lost and is calculated by:

$$B_o(t)-B(t) - R_o(t)-R(t)$$

9. MOE (9) is the surviving BLUE to RED force ratio and is calculated by:

$$\frac{B(t)}{R(t)}$$

10. MOE (10) is the fractional reduction in BLUE to RED force ratio and is calculated by:

$$\frac{\frac{B_o(t)}{R_o(t)} - \frac{B_o(t)}{R_o(t)}}{\frac{B_o(t)}{R_o(t)}}$$

After each battle, the values for all 10 MOE's are both printed and plotted at user specified intervals throughout the length of the battle. This allows for easy analysis of battle outcome.

Model Verification

Verification of the model was accomplished using several techniques. First, the starting force size and the attrition coefficients were set equal for both the BLUE and RED forces, and the engagement was run. This engagement resulted in a draw, with both sides attrited exactly the same. This indicated that the processing portion of the model, that integrates the differential equations, is functioning correctly. Next, the print period (parameter 2) was set to print the results every integration step size, and a line by line hand calculation of the 10 MOE's was conducted to insure that the

results were accurate.

Another verification check that was used, was to adjust the initial model parameters and attrition coefficients to those of a model used by Schaffer (29) and compare the results of the two engagements. The results of these two comparison runs were the same except for a small variation in battle termination time, which was a result of different battle termination rules between the two models.

The model encompasses two different techniques for the integration step size. One is a user specified constant integration step size and the other is a numerical integration algorithm that automatically selects, adjusts and varies the integration step size based on the previous step size. An analysis was made between the two techniques, and it was found that there is a small advantage in using the variable step size in terms of computer execution time.

CHAPTER IV

BATTLE ENGAGEMENT RESULTS

The Base Run

For the purpose of a starting point, a base run of the model was established. This allows a reference point for the analysis of future engagements using parameter variations in the model. In the base run, there is no ammunition constraints or resupply of ammunition, no artillery fire support, and no attrition due to surrenders and desertions. The parameter values of the model for the base run are shown in Figure 7, and were determined from previous experience and Army doctrine.

The starting force size for this run⁶ is 75 BLUE infantry (BINF) and 50 RED infantry (RINF). The engagement lasted 19.75 minutes and the BINF was the winner. Figure 8 is a printout of the results of the base run engagement. The first column of Figure 8 is the time of the engagement, and column's two through eleven list the different MOE values with respect to time. Columns two and three also lists the surviving BLUE and RED forces (BINF and RINF) respectively, at time t . Columns 10 and 11 also lists the remaining total ammunition supply for the BLUE (BAMM) and RED (RAMM) forces respectively, at time t .

1	SOLUTION STEP SIZE	=	.250
2	PRINT PERIOD	=	2.000
3	PLOT PERIOD	=	1.000
4	VARIABLE STEP SIZE MAX %	=	0.000
5	END BATTLE FORCE RATIO	=	.020
6	INITIAL BLUE INFANTRY	=	75.000
7	INITIAL RED INFANTRY	=	50.000
8	GAMMA DENSITY CORRECTION	=	0.000
9	RED AMMO CRIT MIN INV	=	1.000
10	BLUE AMMO CRIT MIN INV	=	1.000
11	BASE EFF BLUE INF FIRE	=	.001
12	BASE EFF RED INF FIRE	=	.001
13	BASE EFF BLUE ARTY FIRE	=	.050
14	BASE EFF RED ARTY FIRE	=	.050
15	BASE RATE FIRE RED INF	=	40.000
16	BASE RATE FIRE BLUE INF	=	40.000
17	RED S+D PER RED CASUAL	=	0.000
18	BLUE S+D PER BLUE CASUAL	=	0.000
19	RED S+D DUE FORCE RATIO	=	0.000
20	BLUE S+D DUE FORCE RATIO	=	0.000
21	INT BLUE AMMO PER MAN	=	10000.000
22	INT RED AMMO PER MAN	=	10000.000
23	DELAY CALL BLUE ARTY	=	10.000
24	DELAY CALL RED ARTY	=	5.000
25	% RED SUPP OF BLUE INF	=	0.000
26	% BLUE SUPP OF RED INF	=	0.000
27	NUM BLUE ARTY TUBES	=	0.000
28	NUM RED ARTY TUBES	=	0.000
29	BLUE INF TV EFF AT ZERO	=	1.000
30	BLUE INF TV EFF TC	=	1.000
31	RED INF TV EFF AT ZERO	=	1.000
32	RED INF TV EFF TC	=	1.000
33	BLUE AMMO CONS FIRE EFF	=	1.000
34	RED AMMO CONS FIRE EFF	=	1.000
35	BLUE RORD FT AMMO/MAN	=	0.000
36	RED RORD FT AMMO/MAN	=	0.000
37	BLUE ORD UP TO AMMO/MAN	=	10000.000
38	RED ORD UP TO AMMO/MAN	=	10000.000
39	BLUE AMMO DELAY TIME	=	0.000
40	RED AMMO DELAY TIME	=	0.000

Figure 7. Parameter Values for Base Run

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	BAMM MOE9	RAMM MOE10
0.00	75. 0.	50. 0.	0.	0.	0.	99.99	99.99	750000. 0.	1.50	500000. 0.00
2.00	71. 4.	44. 6.	5.	12.	-7.	.65	.43	744136. -2.	1.61	496207. -.08
4.00	68. 7.	39. 11.	10.	23.	-13.	.62	.42	738559. -4.	1.76	492873. -.17
6.00	65. 10.	33. 17.	13.	34.	-20.	.60	.40	733233. -7.	1.96	489977. -.30
8.00	62. 13.	28. 22.	17.	44.	-27.	.57	.38	728125. -9.	2.22	487498. -.49
10.00	60. 15.	23. 27.	19.	54.	-34.	.54	.36	723200. -12.	2.60	485422. -.74
12.00	59. 16.	18. 32.	22.	63.	-41.	.52	.34	718428. -15.	3.19	483733. -1.12
14.00	57. 18.	14. 36.	23.	72.	-49.	.49	.32	713777. -19.	4.17	482423. -1.78
16.00	56. 19.	9. 41.	25.	82.	-57.	.45	.30	709219. -22.	6.13	481481. -3.08
18.00	56. 19.	5. 45.	25.	91.	-65.	.42	.28	704723. -26.	11.84	480901. -6.89
19.75	56. 19.	1. 49.	26.	98.	-73.	.39	.26	700818. -30.	55.69	480687. 99.99

Figure 8. Results of Base Run

In addition to the printout of engagement results, the model plots the results using three separate graphs. Figures 9, 10, and 11 are the plots for the base run. The plots produced by the model are all easy to interpret and self-explanatory. The variables to be plotted are defined at the top of the graph and the scales are listed across the top. Time starts at the left top and goes down the left side.

The Gamma Function

As was stated earlier, the value of gamma in Equations (4) determine the form of the Lanchester law. If $\gamma=0$, the equations reduce to the square law. If $\gamma=1$, the equations depict the linear law. For values of between 0 and 1, the equations represent a condition which is neither representative of the square law nor the linear law; but may be representative of some intermediate case. Schmieman (30) refers to this condition as a battle consisting of many small engagements with some engagements satisfying the requirements of the square law, while others, satisfy the requirements of the linear law. Then collectively, the battle should satisfy an intermediate condition, and an optimum value of γ between $\gamma=0$ and $\gamma=1$ should best represent the overall battle. For values of 1, Equations (4) depict the condition referred to by Petterson (26) and Weiss (41) as the "logarithmic law".

In this situation, a side's losses increase with the force committed. Peterson suggests that this probably is due to the fact that the vulnerability of a force, as a target, increases directly with the force committed; but that their effectiveness in delivering firepower increases at a somewhat lesser rate.

To investigate the sensitivity of this model to changes in γ , the base run was replicated 33 times varying γ from zero to one by increments of one tenth for BLUE starting force sizes of 75, 65 and 50; while keeping the RED starting force size constant at 50. The results of these engagements are shown in the table in Figure 12 and the graph in Figure 13. These results confirm Lanchester's (24) original theory of concentration of forces for the square law; that is, that it is more advantageous to increase the number of participants in a battle, than it is to increase by the same factor the effectiveness of weapons.

Surrenders and Desertions

Following some work of Schaffer (29), this model considers the losses due to surrenders and desertions. In Equations (4), surrenders and desertions were indicated by the functions D_B and D_R for the BLUE and RED forces, respectively. These functions are defined as follows:

$$D_B = a_B C_B + b_B \left(\frac{R}{B} - 1 \right)^2$$

$B_0=75, R_0=50$		$B_0=65, R_0=50$		$B_0=50, R_0=50$	
γ	Battle Time (Min)	γ	Battle Time (Min)	γ	Battle Time (Min)
.0	19.75	.0	25.00	.0	97.50
.1	15.00	.1	19.25	.1	80.50
.2	11.50	.2	15.00	.2	67.25
.3	9.00	.3	11.75	.3	57.00
.4	7.00	.4	9.25	.4	48.75
.5	5.75	.5	7.50	.5	42.25
.6	4.50	.6	6.25	.6	37.00
.7	3.75	.7	5.25	.7	32.75
.8	3.00	.8	4.50	.8	29.00
.9	2.75	.9	4.00	.9	26.00
1.0	2.25	1.0	3.50	1.0	23.50

Figure 12. Battle Termination Time as a Function of Gamma and Force Size

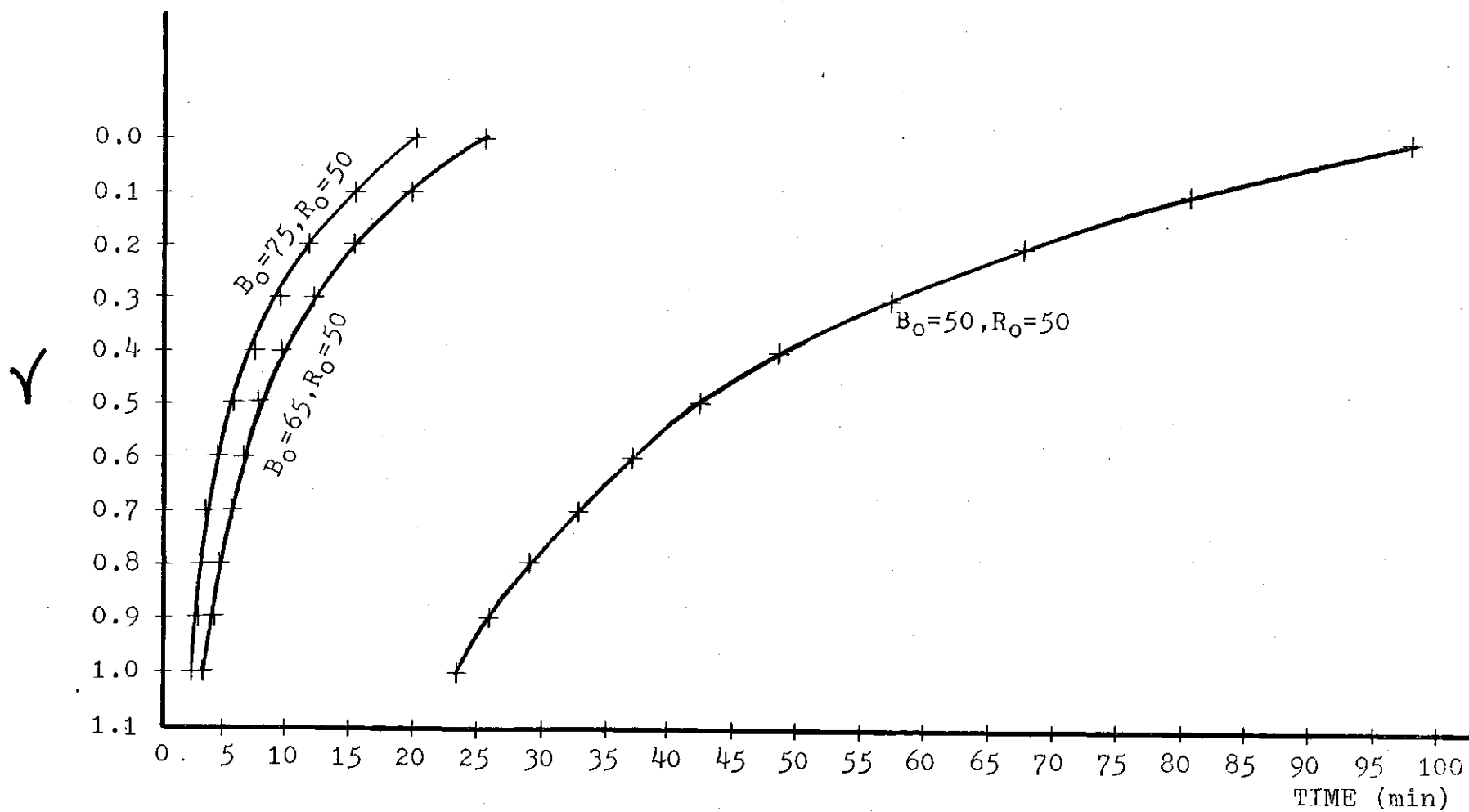


Figure 13. Model Sensitivity to Gamma

$$D_R = a_R C_R + b_R \left(\frac{B}{R} - 1 \right)^2$$

where

a_B = coefficient of BLUE surrenders and desertions per BLUE casualty,

a_R = coefficient of RED surrenders and desertions per RED casualty,

C_B, C_R = casualty rate for BLUE and RED forces, respectively,

b_B, b_R = BLUE and RED surrender and desertion coefficient, respectively, due to force ratio.

As noted above, the rate of friendly surrenders and desertions depends on both the friendly casualty rate and the difference between the friendly force ratio and unity. The coefficients for the surrenders and desertions are what Schaffer refers to as "discipline and morale" of the troops involved in the engagement.

A series of engagements were run by varying both the coefficient for surrenders and desertions and the initial force ratio. The results of these engagements are shown in Figure 14. It is interesting to note that the results of these engagements are the same as those of Schaffer's with the exception of battle termination times, with no variation greater than one minute. This is also another good verification check on the internal processing portion of the model.

(R₀ = 50)

Coefficient		b _B =0, b _R =1		b _B =1, b _R =0		b _B =1, b _R =1	
a _B	a _R	Winner	Time (Min)	Winner	Time (Min)	Winner	Time (Min)
(B ₀ = 75)							
0.0	0.0	BLUE	19.8	BLUE	9.3	BLUE	9.3
0.0	0.5	BLUE	12.3	BLUE	6.8	BLUE	6.8
0.0	1.0	BLUE	9.0	BLUE	5.3	BLUE	5.3
0.5	0.0	BLUE	23.0	BLUE	10.5	BLUE	10.5
0.5	0.5	BLUE	13.3	BLUE	7.3	BLUE	7.3
0.5	1.0	BLUE	9.5	BLUE	5.5	BLUE	5.5
1.0	0.0	BLUE	30.0	BLUE	13.0	BLUE	13.0
1.0	0.5	BLUE	14.5	BLUE	7.8	BLUE	7.8
1.0	1.0	BLUE	10.0	BLUE	6.0	BLUE	6.0

Figure 14. Battle Outcomes as a Result of Surrenders and Desertions

(B₀ = 50)

0.0	0.0	DRAW	DRAW	DRAW	DRAW	DRAW
0.0	0.5	BLUE	23.0	BLUE	15.8	BLUE 23.0 15.8
0.0	1.0	BLUE	15.3	BLUE	11.0	BLUE 15.3 11.0
0.5	0.0	RED	23.0	RED	23.0	RED 15.8 15.8
0.5	0.5	DRAW	DRAW	DRAW	DRAW	DRAW
0.5	1.0	BLUE	18.8	BLUE	13.5	BLUE 18.8 13.5
1.0	0.0	RED	15.3	RED	15.3	RED 11.0 11.0
1.0	0.5	RED	18.8	RED	18.8	RED 13.5 13.5
1.0	1.0	DRAW	DRAW	DRAW	DRAW	DRAW

Figure 14 continued. Battle Outcome as a Result of Surrenders and Desertions

Engagements With Artillery Fire Support

This model has the capability of incorporating supporting artillery fire for the BLUE or RED force, or both. As was stated in Chapter III, the number of artillery tubes per force is specified as a parameter in the model, along with the delay time associated with receiving the fire support, once an engagement starts. Following some work by Willis (45), this model also has artillery suppression factors. These are in terms of percent suppression by RED artillery of surviving BLUE force and percent suppression by BLUE artillery of surviving RED force. The suppression factors effect the rate of attrition due to opposing infantry fire. For example, if RED artillery was deployed against the BLUE force, it would produce BLUE casualties; but it would also cause a decrease in the number of BLUE casualties per minute from RED infantry fire. This happens because the BLUE force is somewhat suppressed, because of the incoming RED artillery rounds, and are less vulnerable as targets to the RED infantry fire as they were before the artillery support began.

Figure 15 is a table of selected battle outcomes with supporting artillery fire. The starting force size was

$$B_0 = 75$$

$$(B_0 = 75, R_0 = 50)$$

NUMBER OF TUBES		DELAY TIME (MIN)		WINNER	TIME (MIN)
BLUE	RED	BLUE	RED		
0	1	10	5	BLUE	28.75
0	2	10	5	RED	50.50
0	3	10	5	RED	25.00
1	1	10	5	BLUE	16.50
1	2	10	5	BLUE	22.25
1	3	10	5	BLUE	35.25
1	4	5	5	RED	24.25
2	1	5	5	BLUE	11.75
2	2	5	5	BLUE	14.25
2	3	5	5	BLUE	18.00
2	4	5	5	RED	45.25
3	1	5	5	BLUE	9.25
3	2	5	5	BLUE	10.50
3	3	5	5	BLUE	12.25

Figure 15. Battle Outcomes With Supporting Artillery

and

$$R_0 = 50$$

The number of tubes per side was varied to see at what point the RED force would win the battle. The results show that with two or more artillery tubes for the RED force and none for the BLUE force, the RED force is the winner. The results also indicate that the RED force wins with four artillery tubes, when the BLUE force has two or less.

Engagements With Ammunition Constraint and Resupply

As was stated in Chapter III, there are six model parameters associated with the ammunition resupply sector. By varying these parameters, there are many combinations available for investigating engagement outcomes associated with ammunition constraint and resupply.

Many engagement runs were conducted by varying the initial force size, number of supporting artillery weapons, and the ammunition sector parameters. Figure 16 lists the results of engagement outcomes with unequal initial force sizes ($B_0=75, R_0=50$) and short delay times for ammunition resupply. Since both forces could get replacement ammunition in a short time span (six minutes for the BLUE force and four minutes for the RED force, all the

$$(B_o = 75, R_o = 50)$$

NUMBER OF ARTY TUBES		WINNER	TIME (MIN)	REASON FOR WIN
BLUE	RED			
0	1	BLUE	30.50	ATTRITION
0	2	RED	48.00	ATTRITION
0	3	RED	25.75	ATTRITION
1	1	BLUE	17.75	ATTRITION
1	2	BLUE	23.75	ATTRITION
1	3	BLUE	37.00	ATTRITION
2	1	BLUE	12.75	ATTRITION
2	2	BLUE	15.50	ATTRITION
2	3	BLUE	19.50	ATTRITION
3	1	BLUE	10.00	ATTRITION
3	2	BLUE	10.75	ATTRITION
3	3	BLUE	12.50	ATTRITION

NOTE: BLUE Ammo Delay Time = 6 minutes

RED Ammo Delay Time = 4 minutes

Initial Ammo Per Man = 300 rounds (BLUE and RED)

Recorder Point Ammo Per Man = 200 rounds (BLUE
and RED)

Figure 16. Engagements With Short Ammunition Resupply
Times

engagements ended because of attrition rather than ammunition shortage. The only time the RED force was the victor was when they possessed two or more supporting artillery weapons and the BLUE force had none.

Figure 17 shows the engagement results when the initial force size was again unequal ($B_0=75, R_0=50$) and the delay times for ammunition resupply were four minutes for the RED force and 20 minutes for the BLUE force. The RED force became the victor in most of the cases, because the BLUE force expended its ammunition supply. The BLUE force was the victor only when they possessed equal or superior artillery fire support.

Figure 18 shows the engagement results with equal initial force size ($B_0=50, R_0=50$) and the delay time for ammunition resupply was again four minutes for the RED force and 20 minutes for the BLUE force. The RED force was the victor in most all cases except when the BLUE force possessed three supporting artillery weapons. Appendix B gives the model outputs (both plots and prints) for the engagements listed in Figure 18.

$$(B_0 = 75, R_0 = 50)$$

NUMBER OF ARTY TUBES		WINNER	TIME (MIN)	REASON FOR WIN
BLUE	RED			
0	1	RED	13.75	AMMUNITION
0	2	RED	17.50	AMMUNITION
0	3	RED	25.75	ATTRITION
1	1	RED	13.50	AMMUNITION
1	2	RED	16.50	AMMUNITION
1	3	RED	22.25	AMMUNITION
2	1	BLUE	13.25	ATTRITION
2	2	BLUE	16.00	ATTRITION
2	3	RED	20.25	AMMUNITION
3	1	BLUE	10.00	ATTRITION
3	2	BLUE	10.75	ATTRITION
3	3	BLUE	12.50	ATTRITION

NOTE: BLUE Ammo Delay Time = 20 minutes

RED Ammo Delay Time = 4 minutes

Initial Ammo Per Man = 300 rounds (BLUE and RED)

Reorder Point Ammo Per Man = 200 rounds (BLUE
and RED)

Figure 17. Engagements With RED Ammunition Resupply Only

$$(B_o = 50, R_o = 50)$$

NUMBER OF ARTY TUBES		WINNER	TIME (MIN)	PERCENT ORIGINAL FORCE LOST		REASON FOR WIN
BLUE	RED			BLUE	RED	
0	1	RED	15.50	70	30	AMMUNITION
0	2	RED	19.50	98	28	ATTRITION
0	3	RED	14.50	98	19	ATTRITION
1	1	RED	14.50	54	64	AMMUNITION
1	2	RED	19.50	78	65	AMMUNITION
1	3	RED	23.00	98	60	ATTRITION
2	1	RED	14.00	45	83	AMMUNITION
2	2	RED	17.75	65	85	AMMUNITION
2	3	RED	25.25	83	86	AMMUNITION
3	1	BLUE	13.75	39	94	ATTRITION
3	2	BLUE	16.75	57	96	ATTRITION
3	3	BLUE	21.75	71	98	ATTRITION

NOTE: BLUE Ammo Delay Time = 20 minutes

RED Ammo Delay Time = 4 minutes

Initial Ammo Per Man = 300 rounds (BLUE and RED)

Reorder Point Ammo Per Man = 200 rounds (BLUE
and RED)

Figure 18. Equal Force Size Engagements With RED Ammunition Resupply Only

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This thesis has presented a general digital model for use in solving a class of Lanchester-type warfare models. The model includes such factors as time delays, effects of surrenders and desertions, and artillery fire support. In addition, it has extended the Lanchester theory by the addition of ammunition constraints and re-supply.

Over 2,000 battle engagements were executed with the model. The results proved both interesting and informative. The model was very sensitive in the choice of gamma (the parameter associated with switching from the square law to the linear law, as gamma is varied from zero to one) for initial unequal opposing force sizes; but less sensitive as the ratio of the initial force sizes approached unity. This supports Lanchester's idea of concentration of forces, but differs somewhat from the results of Schmieman's (30) study.

A series of engagements were conducted with ammunition constraints and resupply, including engagements with and without artillery fire support. Battle outcomes

were listed for various cases and the results showed the effect of ammunition delay times. Even though one side was initially outnumbered 1.5 to 1, the smaller force size could be the victor by having a quick effective resupply of ammunition. The results could be examined to determine the trade off between ammunition resupply times the number of supporting artillery weapons.

The model produces a convenient display of results through both printing and plotting that are easy to understand and interpret. This gives the capability for a variety of analyses of the engagement's results.

Recommendations

This model offers the potential for further study of Lanchester-type warfare equations. With 40 parameters available for user specified values, it provides an almost infinite number of possibilities for different engagements that could be studied. Additional research is needed in the area of experimental design of engagement runs with this model. Clearly, a 15 parameter variation each at five levels, for example, would require 5^{15} separate runs. In order to optimize for the minimum number of required runs, some kind of effective experimental design would have to be developed. The potential for factorial design or response surface methodology should be investigated by someone knowledgeable in these areas.

Because of the structure of this model, it lends itself to a fairly easy adaption to a syndetic analog-digital computer that could be used to investigate further, this class of equations at a substantially lower cost. The implementation of this model to an analog-digital computer would require four integrations (BLUE infantry, RED infantry, BLUE ammunition, and RED ammunition), several function generators (e.g. ammunition constraints, etc.), and 40 potentiometers for the parameters. The combination of fast analog computation time, coupled with an automated display, could be very valuable. For example, it could allow for multiple MOEs through time on the same display, or a comparative display of MOEs versus a range of parameters. This type of capability would certainly allow an easier analysis of the behavior of the model.

Although this thesis investigated the behavior of the model for small scale encounters, it could certainly be expanded for use with larger force size engagements, to include the addition of the replacement of forces, based on the attrition rates and length of engagements.

Another recommendation would be to investigate engagement outcomes between two battles, using the same parameter values for each case, but having different battle termination rules. For example, let one engagement end when one of the forces loses all of its men;

then, run the engagement again with a termination rule that says, "stop the engagement when one force has 80% of its troops remaining." In other words, the commander is only willing to suffer a 20% loss before he withdraws from the engagement. By doing a comparison of this nature, the results may show different winners, because of the length of the engagements and the ammunition constraint functions.

Beyond academic interests to investigate parameter sensitivity questions still open (e.g. delay times, resupply policies, etc.), a large scale model such as this can be used to address real problems. The design of United States Army combat units in Europe, for example, is an obvious application. The procedure would include estimating the model parameters to optimize them in order to respond to potential Soviet threat, given that fairly accurate estimates were available on USSR unit size, force mixers, weapon capabilities, discipline and morale, etc.

APPENDIX A

LISTING OF SYMBOLS AND NOTATIONS

$\frac{dB}{dt}, \frac{dR}{dt}$ = the attrition rate of the BLUE and RED forces, respectively, with respect to time,

B_0, R_0 = the number of BLUE and RED forces at time $t=0$,

J, K = the coefficient of effectiveness for the BLUE and RED forces, respectively,

N = the number of duels per unit of time,

$B(t)^\gamma, R(t)^\gamma$ = the form of law, where $\gamma=0$ is the square law and $\gamma=1$ is the linear law,

S_B, S_R = the suppression factor for BLUE and RED forces due to opposing fire,

$B(t), R(t)$ = the number of surviving BLUE and RED forces at time t ,

r_B, r_R = the rate of infantry fire (rounds per minute) for BLUE and RED forces,

D_B, D_R = the losses due to surrenders and desertions for the BLUE and RED forces,

- a_B = the coefficient of BLUE surrenders and desertions per BLUE casualty,
- a_R = the coefficient of RED surrenders and desertions per RED casualty,
- C_B, C_R = the casualty rate for BLUE and RED forces,
- b_B, b_R = the BLUE and RED surrender and desertion coefficient due to the force ratio,
- α_B, α_R = the coefficient of effectiveness for the supporting artillery fire for the BLUE and RED forces,
- W_B, W_R = the number of supporting artillery weapons for the BLUE and RED forces,
- $f(A_B), f(A_R)$ = the ammunition constraint function for the BLUE and RED forces,
- d = the ratio of remaining ammunition per man,
- e = the critical ratio of ammunition per man,

β = the battle ending force ratio, and

B_0, R_0 = initial force size at $t=0$.

APPENDIX B

FORTRAN LISTING OF GENERAL

LANCHESTER-TYPE

WARFARE MODEL

PROGRAM MAIN(INPUT,OUTPUT,SPOOL,TAPE5=INPUT,TAPE6=OUTPUT,
1TAPE99=SPOOL)

C
C
C
C

*****MASTER CONTROL LINK FOR GENERAL LANCHESTER MODEL*****

IMPLICIT REAL(A-Z)
INTEGER NRUN,1,J,INDEX,NPARN,IFLAG(040),ITYPE
INTEGER IRUN,ISTOP,IC,ICP,ISCAL,INOTE
DIMENSION PARN(040),ORIG(040),CP(040)
DIMENSION TEXT(040,4)
INTEGER NSPOOL,LETTER(15),MESAG(11),NAME(15)
REAL SCALE(15,2)
COMMON / / NRUN,NSPOOL,LETTER,SCALE,MESAG,NAME
COMMON /PRM/DT,FRTPER,PLTPER,VARSTP,ENDFR,BINFZ,RINFZ,GAMMA,RAMCON
COMMON /PRM/BAMCON,BINFBE,RINFBE,BOTHBE,ROTHBE,RSHOTB,BSHOTB
COMMON /PRM/RSDPFC,BSDPFC,RSDDFR,BSDDFR,BAPIN,RAPIN,BOTHST,ROTHST
COMMON /PRM/RBBF,BSRF,BOTH,ROTH,BITVEZ,BITVES,RITVEZ,RITVES
COMMON /PRM/BAMCEF,RAMCEF,BRTAP,RETAP,BRUPT,RRUPT,BOEL,RDEL
EQUIVALENCE (PARN(1),DT)
DATA IRUN,ISTOP,ICP,IC/4HRUN ,4HSTOP,4HCP ,4HC /
DATA INOTE,ISCAL/4HNOTE,4HSCAL/
DATA(TEXT(1,J),J=1,4)/6HSOLUT1,6HON STE,6HP SIZE,6H /
DATA(TEXT(2,J),J=1,4)/6HPRINT ,6HPERIOD,6H ,6H /
DATA(TEXT(3,J),J=1,4)/6HPLOT F,6HERIOD ,6H ,6H /
DATA(TEXT(4,J),J=1,4)/6HVARIAB,6HLE STE,6HP SIZE,6H MAX %/
DATA(TEXT(5,J),J=1,4)/6HEND BA,6HTLE F,6HORCE R,6HATIO /
DATA(TEXT(6,J),J=1,4)/6HINITIA,6HL BLUE,6H INFAN,6HTRY /
DATA(TEXT(7,J),J=1,4)/6HINITIA,6HL RED ,6HINFANT,6HRY /
DATA(TEXT(8,J),J=1,4)/6HGAMMA ,6HDENSIT,6HY CORR,6HECTION/
DATA(TEXT(9,J),J=1,4)/6HRED AM,6HMO ORI,6HT MIN ,6HINV /
DATA(TEXT(10,J),J=1,4)/6HBLUE A,6HMO CR,6HT MIN,6H INV /

DATA(TEXT(11,J),J=1,4)/6HBASE E,6HFF BLU,6HE INF ,6HFIRE /
 DATA(TEXT(12,J),J=1,4)/6HBASE E,6HFF RED,6H INF F,6HIRE /
 DATA(TEXT(13,J),J=1,4)/6HBASE E,6HFF BLU,6HE ARTY,6H FIRE /
 DATA(TEXT(14,J),J=1,4)/6HBASE E,6HFF RED,6H ARTY ,6HFIRE /
 DATA(TEXT(15,J),J=1,4)/6HBASE R,6HATE FI,6HRE RED,6H INF /
 DATA(TEXT(16,J),J=1,4)/6HBASE R,6HATE FI,6HRE BLU,6HE INF /
 DATA(TEXT(17,J),J=1,4)/6HRED S+,6HD PER ,6HRED CA,6HSUAL /
 DATA(TEXT(18,J),J=1,4)/6HBLUE S,6H+D PER,6H BLUE ,6HCASUAL/
 DATA(TEXT(19,J),J=1,4)/6HRED S+,6HD DUE ,6HFORCE ,6HRATIO /
 DATA(TEXT(20,J),J=1,4)/6HBLUE S,6H+D DUE,6H FORCE,6H RATIO/
 DATA(TEXT(21,J),J=1,4)/6HINT BL,6HUE AMM,6HO PER ,6HMAN /
 DATA(TEXT(22,J),J=1,4)/6HINT RE,6HD AMMO,6H PER M,6HAN /
 DATA(TEXT(23,J),J=1,4)/6HDELAY ,6HCALL B,6HBLUE AR,6HTY /
 DATA(TEXT(24,J),J=1,4)/6HDELAY ,6HCALL R,6HED ART,6HY /
 DATA(TEXT(25,J),J=1,4)/6H% RED ,6HSUPP O,6HF BLUE,6H INF /
 DATA(TEXT(26,J),J=1,4)/6H% BLUE,6H SUPP ,6HCF RED,6H INF /
 DATA(TEXT(27,J),J=1,4)/6HNUM BL,6HUE ART,6HY TUBE,6HS /
 DATA(TEXT(28,J),J=1,4)/6HNUM RE,6HD ARTY,6H TUBES,6H /
 DATA(TEXT(29,J),J=1,4)/6HBLUE I,6HNF TV ,6HEFF AT,6H ZERO /
 DATA(TEXT(30,J),J=1,4)/6HBLUE I,6HNF TV ,6HEFF TC,6H /
 DATA(TEXT(31,J),J=1,4)/6HRED IN,6HF TV E,6HFF AT ,6HZERO /
 DATA(TEXT(32,J),J=1,4)/6HRED IN,6HF TV E,6HFF TC ,6H /
 DATA(TEXT(33,J),J=1,4)/6HBLUE A,6HMMO CO,6HNS FIR,6HE EFF /
 DATA(TEXT(34,J),J=1,4)/6HRED AM,6HMO CON,6HS FIRE,6H EFF /
 DATA(TEXT(35,J),J=1,4)/6HBLUE R,6HORD PT,6H AMMO/,6HMAN /
 DATA(TEXT(36,J),J=1,4)/6HRED RO,6HRD PT ,6HAMMO/M,6HAN /
 DATA(TEXT(37,J),J=1,4)/6HBLUE O,6HRD UP ,6HTO AMM,6HO/MAN /
 DATA(TEXT(38,J),J=1,4)/6HRED OR,6HD UP T,6HO AMMO,6H/MAN /
 DATA(TEXT(39,J),J=1,4)/6HBLUE A,6HMMO DE,6HLAY TI,6HME /
 DATA(TEXT(40,J),J=1,4)/6HRED AM,6HMO DEL,6HAY TIM,6HE /

```

C          *****SPECIFICATION OF BASE PARAMETERS*****
C
C          *****A FEW WILL BE CHANGED IN EACH RERUN*****
C
C          NRUN=0
C              RUN NUMBER
C          NPARM=040
C              NUMBER OF PARAMETERS
C
C          MESAG(1)=6HLANCHE
C          MESAG(2)=6HSTER 1
C          MESAG(3)=6HNFANTR
C          MESAG(4)=6HY MODE
C          MESAG(5)=6HL W/AR
C          MESAG(6)=6HTY,RES
C          MESAG(7)=6HUP,DEL
C          MESAG(8)=6HAY,TV
C          MESAG(9)=6HCOEFF,
C          MESAG(10)=6HINV CC
C          MESAG(11)=6HNST
C
C          SPECIFICATION OF CONTROL PARAMETERS FOR PROGRAM OPERATION
C          DT=.25
C          FIXED STEP SIZE (EULER INTEGRATION)
C          VARSTP=0.
C          VARIABLE STEP SIZE INDICATOR . IF NON-ZERO STEP WILL
C          BE SET OF GUARANTEE NO MORE THAN VARSTP PERCENT
C          CHANGE PER INTERVAL IN EACH STATE VARIABLE
C          PRTPER=2
C          PRINT INTERVAL
C          PLTPER=1
C          PLOT INTERVAL

```

C
C

ENDFR=.02

C
C
C

END OF BATTLE OCCURS AT THIS FORCE RATIO

SPECIFICATION OF CONSTANTS

BINFZ=75.

RINFZ=50.

C

INITIAL VALUES FOR INFANTRY FORCES

BAPIN=10000.

C

INITIAL ROUNDS PER MAN BLUE INF

RAPIN=10000.

C

INITIAL ROUNDS PER MAN RED INF

GAMMA=0.

C

DENSITY CORRECTION, ZERO FOR SQUARE LAW

RAMCON=1.

BAMCON=1.

C

TIME CONSTANTS FOR AMMO CONSTRAINT ON FIRE RATES

BINFBE=.001

RINFBE=.001

C

BASE EFFECTIVENESS--PROB KILL PER ROUND GIVEN PLENTY AMMO

BOTHBE=.05

ROTHBE=.05

C

BASE AUXILLARY EFFECTIVENESS

RSHOTB=40.

BSHOTB=40.

C

BASE RATE OF FIRE PER SOLDIER PER TIME

C

BOTHST=10.

ROTHST=5.

C

STARTING TIMES OF AUXILLARY FIRE

RSDFRC=0.

BSDFBC=0.
 C SURRENDERS AND DESERTIONS PER CASUALTY
 RSDDFR=0.
 BSDDFR=0.
 C SURRENDERS AND DESERTIONS DUE TO FORCE RATIO IMBALANCE
 RSBF=0.
 BSRF=0.
 C FRACTION SUPPRESSION OF INF FIRE DUE TO OPPOSING AUX FIRE
 C
 BOTH=0.
 ROTH=0.
 C AUXILLARY FIRE ELEMENTS (E.G. TUBES)
 C
 BITVEZ=1.
 RITVEZ=1.
 C RELATIVE INF. FIRE TIME VARYING EFF. AT TIME ZERO
 BITVES=1.
 RITVES=1.
 C RELATIVE INF. FIRE TIME VARYING EFF. TIME CONSTANT
 BAMCEF=1.0
 RAMCEF=1.0
 C MAX. FACTOR INCREASE IN FIRE EFFECT WITH SMALL AMMO INV
 BRTAP=0.
 RRTAP=0.
 C RE-ORDER POINTS FOR AMMO WHEN AMMO/MAN THIS VALUE
 BRUPT=BAPIN
 RRUPT=RAPIN
 C RE-ORDER UP TO THIS AMOUNT AMMO/MAN
 BCEL=0.
 RDEL=0.
 C DELAY TIMES FOR RECEIVING AMMO RESUPPLY ORDERS
 C

C STANDARD SCALES FOR PLOT OUTPUT

```
DO 300 I=1,15
300  SCALE(I,1)=0.
      XX=AMAX1(BINFZ,RINFZ)
      SCALE(1,2)=XX
      SCALE(2,2)=XX
      YY=AMAX1(BAPIN*BINFZ,RAPIN*RINFZ)
      SCALE(3,2)=YY
      SCALE(4,2)=YY
      SCALE(5,2)=BINFZ
      SCALE(6,2)=RINFZ
      SCALE(7,2)=100.
      SCALE(8,2)=100.
      SCALE(9,1)=-100.
      SCALE(9,2)=100.
      SCALE(10,2)=10.
      SCALE(11,2)=10.
      SCALE(12,1)=-XX
      SCALE(12,2)=XX
      SCALE(13,2)=10.
      SCALE(14,1)=-3.
      SCALE(14,2)=1.
      NAME(1)=6H BINF
      NAME(2)=6H RINF
      NAME(3)=6H BMM
      NAME(4)=6H RMM
      NAME(5)=6HMOE(1)
      NAME(6)=6HMOE(2)
      NAME(7)=6HMOE(3)
      NAME(8)=6HMOE(4)
      NAME(9)=6HMOE(5)
      NAME(10)=6HMOE(6)
```

```
NAME(11)=6HMOE(7)
NAME(12)=6HMOE(8)
NAME(13)=6HMOE(9)
NAME(14)=6H MOE10
```

C
C

```
      CANNOT USE DATA STATEMENT FOR COMMONED ARRAY
```

```
LETTER(1)=1HB
LETTER(2)=1HR
LETTER(3)=1HY
LETTER(4)=1HZ
LETTER(5)=1H1
LETTER(6)=1H2
LETTER(7)=1H3
LETTER(8)=1H4
LETTER(9)=1H5
LETTER(10)=1H6
LETTER(11)=1H7
LETTER(12)=1H8
LETTER(13)=1H9
LETTER(14)=1H0
```

C
C
C
C
C
C

```
*****END OF PARAMETER SPECIFICATION*****
```

```
      STORE VALUES IN VECTOR ORIG
```

```
DO 11 I=1,NPARAM
IFLAG(I)=0
CF(I)=PARM(I)
11 ORIG(I)=PARM(I)
```

C
C

```

C      LIST MASTER PARMETERS AT TOP OF RUN
C
      WRITE(6,120) MESAG
120  FORMAT(*1*//20X,*ORIGINAL PARAMETERS*/5X,11A6//)
      DO 121 I=1,NPARM
121  WRITE(6,122) I,(TEXT(I,J),J=1,4),ORIG(I)
122  FORMAT(10X,I2,2X,4A6,* = *,F11.3)
C
C      SETUP A NEW PARAMETRIC RUN
C
      3  NRUN=NRUN+1
      WRITE(6,113) NRUN,MESAG
113  FORMAT(*1*//20X,*RUN NUMBER *,I4/5X,11A6//)
      IF(NRUN.EQ. 1) GO TO 16
C
C      RESTORE ORIGINAL VALUES OF PARAMETERS
      DO 10 I=1,NPARM
10  PARM(I)=CP(I)
C
C      LIST PERMANENT CHANGES CURRENTLY ACTIVE
      DO 15 I=1,NPARM
      IF(IFLAG(I).EQ. 0) GO TO 15
      WRITE(6,111)
111  FORMAT(//20X,*PERMANENT PARAMETER CHANGES*/)
      GO TO 13
      15  CONTINUE
      GO TO 17
      13  DO 14 I=1,NPARM
      IF(IFLAG(I).EQ. 0) GO TO 14
      WRITE(6,101) I,(TEXT(I,J),J=1,4),PARM(I),ORIG(I)
      14  CONTINUE
C

```



```

17  WRITE(6,108)
108  FORMAT(/20X,*, NEW PARAMETER CHANGES */)
C
16  READ(5,998) ITYPE,INDEX,VALUE,UPPER
998  FORMAT(A4,I2,2F10.0)
    IF(ITYPE .EQ. INOTE) GO TO 16
    IF(ITYPE .EQ. IRUN) GO TO 2
    IF(ITYPE .EQ. ISTOP) GO TO 1
    IF(ITYPE .EQ. ISCAL) GO TO 30
C
    IF(INDEX .GT. 0 .AND. INDEX .LE. NPARM) GO TO 20
    WRITE(6,102) INDEX
102  FORMAT(5X,*, ILLEGAL PARAMETER NUMBER* I5)
    GO TO 16
C
20  IF(ITYPE .EQ. ICP) GO TO 22
C
    TEMPORARY CHANGE
    IFLAG(INDEX)=0
    CF(INDEX)=ORIG(INDEX)
    PARM(INDEX)=VALUE
    GO TO 23
C
    PERMANENT CHANGE
22  IFLAG(INDEX)=1
    CF(INDEX)=VALUE
    PARM(INDEX)=VALUE
C
23  WRITE(6,101) INDEX, (TEXT(INDEX,J), J=1,4), PARM(INDEX), ORIG(INDEX)
101  FORMAT(5X,*,PARAMETER *,I3,2X,4A6/25X, *,NEW VALUE *,F11.3,
1*  REPLACES *,F11.3/)
C

```

```

C      AUTOMATIC CHANGE OF BINF/RINF AND BMM/RRM SCALES
      IF(INDEX .NE. 6 .AND. INDEX .NE. 7) GO TO 500
C      NEW BINFZ OR RINFZ
      XX=AMAX1(BINFZ,RINFZ)
      SCALE(1,2)=XX
      SCALE(2,2)=XX
      GO TO 501
C
500      IF(INDEX .NE. 21 .AND. INDEX .NE. 22) GO TO 16
C      CHANGE BMM/RRM SCALES
501      YY=AMAX1(BINFZ*BAPIN,RINFZ*BAPIN)
      SCALE(3,2)=YY
      SCALE(4,2)=YY
      GO TO 16
C
C      SCALE CHANGES--THESE CHANGES PERMANENT--
30      IF(INDEX .LE. 1 .OR. INDEX .GT. 14) GO TO 33
      WRITE(6,31) INDEX,NAME(INDEX),VALUE,UPPER,(SCALE(INDEX,I),I=1,2)
31      FORMAT(5X,*NEW PERMANENT SCALE SET FOR *,I3,1X,A6/10X,*(*,
1      E9.2,* ,*,E9.2,*) REPLACES (*,E9.2,* ,*,E9.2,* )*)
      SCALE(INDEX,1)=VALUE
      SCALE(INDEX,2)=UPPER
      GO TO 16
33      WRITE(6,34) INDEX
34      FORMAT(10X,*BAD SCALE INDEX* ,I10)
      GO TO 16
1      STOP
C
2      CALL LANCH
      IF(NSPOOL .EQ. 0) GO TO 3
      CALL PLOT0(101,102,114,103,104,0)
      CALL PLOT0(105,106,107,108,0,0)

```

```

C      CALL PLOTG(109,110,111,112,113,0)
C
C      GO TO 3
C
C      END
C
      SUBROUTINE LANCH
      IMPLICIT REAL(A-Z)
      INTEGER NRUN,NSPOOL,LETTER(15),MESAG(11),NAME(15)
      REAL SCALE(15,2)
      INTEGER II,JJ,JSWIT
      DIMENSION MOE(10),SUPPLY(4,50,2),SARRIV(4),SONORD(4)
      COMMON / / NRUN,NSPOOL,LETTER,SCALE,MESAG,NAME
      COMMON /PRM/DT,PRTPER,PLTPER,VARSTP,ENDFR,BINFZ,RINFZ,GAMMA,RAMCON
      COMMON /PRM/BAMCON,BINFBE,RINFBE,BOTHBE,ROTHBE,RSHOTB,BSHOTB
      COMMON /PRM/RSDFR,BSDFBC,RSDDFR,BSDDFR,BAPIN,RAPIN,BOTHST,ROTHST
      COMMON /PRM/RSBF,BSRF,BOTH,ROTH,BITVEZ,BITVES,RITVEZ,RITVES
      COMMON /PRM/BAMCEF,RAMCEF,BRTAP,RRTAP,BRUPT,RRUPT,BDEL,RCEL
C
C
C      JSWIT=0
C      END OF RUN CONDITION INDICATOR
      NSPOOL=0
C      NUMBER OF LINES OF PLOT SPOOLED
      REWIND 99
      NXTPRT=0.
      IF(PRTPER.EQ.0.) NXTPRT=99999.
C      NEXT PRINT TIME
      NXTPLT=0.
      IF(PLTPER.EQ.0.) NXTPLT=99999.
C      NEXT PLOT TIME

```

```

C
C
C      INITIALIZATION OF STATE VARIABLES
RINF=RINFZ
BINF=BINFZ
FZERO=BINFZ/RINFZ
C      INITIAL FORCE RATIO
BAMM=BAPIN*BINFZ
RAMM=RAPIN*RINFZ
C      INITIAL AMMUNITION INVENTORIES
TIME=0.
C
C      INITIALIZE SUPPLY VECTORS
DO 500 II=1,4
SONORD(II)=0.
DO 500 JJ=1,50
SUPPLY(II,JJ,1)=0.
C      AMOUNT OF RESUPPLY
SUPPLY(II,JJ,2)=99999.
C      TIME OF SUPPLY
500 CONTINUE
C
WRITE(6,103)
103 FORMAT(/3X,*TIME*,2X,*BINF*,2X,*RINF*,32X,*BAMM*,9X,*RAMM*/
17X,* MOE1 MOE2 MOE3 MOE4 MOE5 MOE6 MOE7 MOE8 MOE9 MOE10*
2)
GO TO 210
C
C      CALCULATE DERIVATIVES
C      EXOGENOUS COMPONENTS OF DERIVATIVES
1 CONTINUE
C

```

```

C
C      ENDOGENOUS COMPONENTS OF DERIVATIVES
RABOTH=BOTHBE+BOTH*RINF*STEP(1.,BOTHST-TIME)
BAROTH=ROTHBE*ROTH*BINF*STEP(1.,ROTHST-TIME)
RSBINF=STEP(RSBF,ROTHST-TIME)
BSRINF=STEP(BSRF,BOTHST-TIME)
C      ---RATE OF ATTRITION DUE TO AUXILLARY FIRE
BFIRE=BINF*FSHOTT(BAMM/BINF,BAMCON*BSHOTB)*BSHOTB
RFIRE=RINF*FSHOTT(RAMM/RINF,RAMCON*RSHOTB)*RSHOTB
C      --- ROUNDS FIRED PER SOLIDER PER TIME
BARINF=RINFBE*(1.-BSRINF)*(BINF**GAMMA)*RFIRE*
1  FAAIM(RAMM/RINF,RAMCEF,RAMCON)*TVEFF(TIME,RITVEZ,RITVES)
RABINF=BINFBE*(1.-RSBINF)*(RINF**GAMMA)*BFIRE*
1  FAAIM(BAMM/BINF,BAMCEF,BAMCON)*TVEFF(TIME,BITVEZ,BITVES)
C      --- RATE OF ATTRITION DUE TO OPPOSING INFANTRY FIRE
RINFSD=RSDPRC*RABINF
IF(BINF/RINF .LE. 1.) GO TO 701
RINFSD=RINFSD+RSDCFR*(BINF/RINF-1.)**2
701  BINFSDBSDPBC*BARINF
IF(RINF/BINF .LE. 1.) GO TO 702
BINFSDBINFSDBSDCFR*(RINF/BINF-1.)**2
702  CONTINUE
C      ---LOSSES FROM SURRENDERS AND DESERTIONS
DO 590 II=1,4
590  SARRIV(II)=0.
C      ---RESUPPLY ARRIVING FOR BINF,RINF,BAMM,RAMM
NEXTST=AMIN1(SUPPLY(1,1,2),SUPPLY(2,1,2),SUPPLY(3,1,2),
1  SUPPLY(4,1,2))
C      --TIME OF NEXT RESUPPLY
C
C
C      UPDATE STATE VARIABLES

```

```

C
C      GET TOTAL DERIVATIVES
C
DRINF=-RABINF-RINFSD-RABOTH
DBINF=-BARINF-BINFSD-BAROTH
DBAMM=-BFIRES
DRAMM=-RFIRES
C
C      SELECT INTEGRATION STEP SIZE
C      IF(VARSTP .EQ. 0.) GO TO 801
C
C      VARIABLE STEP SIZE
MAXPC=AMAX1(ABS(DRINF/RINF),ABS(DBINF/BINF),ABS(DRAMM/RAMM),
1ABS(DBAMM/BAMM))
QDT=VARSTP/MAXPC
IF(QDT .LT. .005) QDT=.005
IF(TIME+QDT .GT. NXTPLT) QDT=NXTPLT-TIME
IF(TIME+QDT .GT. NXTPRT) QDT=NXTPRT-TIME
C      CHECK FOR START-UP OF AUXILLARY FIRE
IF(TIME .LT. BOTHST .AND. TIME+QDT .GT. BOTHST) QDT=BOTHST-TIME
IF(TIME .LT. ROTHST .AND. TIME+QDT .GT. ROTHST) QDT=ROTHST-TIME
C
C      CHECK FOR RESUPPLY
IF(NEXTST .EQ. 99999.) GO TO 802
IF(TIME+QDT .LT. NEXTST) GO TO 802
C
C      YES THERE IS TO BE RESUPPLY
QDT=NEXTST-TIME
DO 581 II=1,4
IF(SUPPLY(II,1,2) .GT. NEXTST) GO TO 581
SARRIV(II)=SUPPLY(II,1,1)
SONORD(II)=SONCRD(II)-SARRIV(II)

```

```

      DO 583 JJ=1,49
      SUPPLY(II,JJ,1)=SUPPLY(II,JJ+1,1)
583    SUPPLY(II,JJ,2)=SUPPLY(II,JJ+1,2)
      SUPPLY(II,50,1)=0.
      SUPPLY(II,50,2)=99999.
581    CONTINUE
      GO TO 802
C
C      FIXED STEP SIZE
801    QDT=DT
      IF(NEXTST .EQ. 99999.) GO TO 802
C      RESUPPLY ARRIVING--YES IF TIME IS WITHIN DT/2.
      DO 560 II=1,4
      IF( (ABS(SUPPLY(II,1,2))-TIME-QT) .GT. DT/2.) GO TO 560
C      II RESUPPLY
      SARRIV(II)=SUPPLY(II,1,1)
      SONORD(II)=SONCRD(II)-SARRIV(II)
      DO 564 JJ=1,49
      SUPPLY(II,JJ,1)=SUPPLY(II,JJ+1,1)
564    SUPPLY(II,JJ+1,2)=SUPPLY(II,JJ+1,2)
      SUPPLY(II,50,1)=0.
      SUPPLY(II,50,2)=99999.
560    CONTINUE
C
C      INTEGRATE
802    BINF=BINF+(QDT)*DBINF+SARRIV(1)
      RINF=RINF+(QDT)*DRINF+SARRIV(2)
      BAMB=BAMB+(QDT)*DBAMB+SARRIV(3)
      RAMM=RAMM+(QDT)*DRAMM+SARRIV(4)
      TIME=TIME+QDT
C
C

```

```

C      RESUPPLY SECTOR
C
C
C      CHECK BLUE AMMO RESUPPLY
C      IF(BAMM/BINF+SONORD(3)/BINF .GT. BRTAP) GO TO 1000
C      RE-ORDER FOR BLUE
      DO 1010 JJ=1,50
      IF(SUPPLY(3,JJ,1) .NE. 0) GO TO 1010
      SUPPLY(3,JJ,1)=BRUPT*BINF-BAMM
      SUPPLY(3,JJ,2)=BDEL+TIME
      SONORD(3)=BRUPT*BINF-BAMM
      GO TO 1000
1010    CONTINUE
1012    WRITE(6,1011)
1011    FORMAT(* TOO BAD, RESUPPLY VECTOR FILLED*)
      STOP
C
C      OF, CHECK RED RESUPPLY
1000    IF(RAMM/RINF + SONORD(4)/RINF .GT. RRTAP) GO TO 1030
      DO 1020 JJ=1,50
      RE-ORDER RED AMMO
      IF(SUPPLY(4,JJ,1) .NE. 0) GO TO 1020
      SUPPLY(4,JJ,1)=RRUPT*RINF-RAMM
      SUPPLY(4,JJ,2)=RDEL+TIME
      SONORD(4)=SONORD(4)+RRUPT*RINF-RAMM
      GO TO 1030
1020    CONTINUE
      GO TO 1012
C
1030    CONTINUE
C      CALCULATE AND PRINT MOES
C

```



```

C      TIME TO CALCULATE?
C      WATCH OUT FOR DIFFERENT RULES FIXED AND VARIABLE STEP
IF(VARSTP .EQ. 0.) GO TO 773
C      ---HERE IS TEST FOR VARIABLE STEP INTEGRATION
IF(TIME .NE. NXTPRT .AND. TIME .NE. NXTPLT) GO TO 200
GO TO 210
C
773 IF( ABS(TIME-NXTPRT) .GT.DT/2. .AND.
1 ABS(TIME-NXTPLT) .GT. DT/2.) GO TO 200
C      IT IS TIME TO CALCULATE AND PRINT OUTPUT
210 MOE(1)=BINFZ-BINF
C      NUMBER OF BLUE INFANTRY LOST
MOE(2)=RINFZ-RINF
C      NUMBER OF RED INFANTRY LOST
MOE(3)=100.*MOE(1)/BINFZ
C      PERCENT BLUE INFANTRY LOST
MOE(4)=100.*MOE(2)/RINFZ
C      PERCENT RED INFANTRY LOST
MOE(5)=MOE(3)-MOE(4)
C      DIFFERENCE IN PERCENT LOST
IF(MOE(2) .LT. .001) MOE(6)=99.99
IF(MOE(2) .GE. .001) MOE(6)=MOE(1)/MOE(2)
C      RATIO OF NUMBER INFANTRY LOST
IF(MOE(4) .LT. .001) MOE(7)=99.99
IF(MOE(4) .GE. .001) MOE(7)=MOE(3)/MOE(4)
C      RATIO OF PERCENT ORIGINAL INFANTRY LOST
MOE(8)=MOE(1)-MOE(2)
C      DIFFERENCE IN NUMBER OF INFANTRY LOST
IF(RINF .LT. 1.) MOE(9)=BINF
IF(RINF .GE. 1.) MOE(9)=BINF/RINF
C      SURVIVING FORCE RATIO
IF(RINF .LT. 1.) MOE(10)=99.99

```

```

      IF(RINF .GT. 1.) MOE(10)=(FZERO-MOE(9))/FZERO
C      FRACTIONAL REDUCTION IN FORCE RATIO
C
      IF(JSWIT .EQ. 1) GO TO 212
      IF(PRTPER .EQ. 0.) GO TO 220
      IF(VARSTP .EQ. 0.) GO TO 774
      IF(TIME .NE. NXTPRT) GO TO 220
      GO TO 230
774 IF( ABS(TIME-NXPRT) .GT. DT/2.) GO TO 220
230 WRITE(6,100) TIME,BINF,RINF,BAMM,RAMM
100 FORMAT(/1X,F6.2,2F6.0,24X,F12.0,F13.0)
      WRITE(6,201) MOE
      NXTPRT=TIME+PRTPER
C
220 IF(PLTPER .EQ. 0.) GO TO 200
      IF(VARSTP .EQ. 0.) GO TO 775
      IF(TIME .NE. NXTPLT) GO TO 200
      GO TO 231
775 IF( ABS(NXTPLT-TIME) .GT. DT/2.) GO TO 200
231 NSPOOL=NSPOOL+1
      WRITE(99) TIME,BINF,RINF,BAMM,RAMM,(MOE(II),II=1,10)
      NXTPLT=TIME+PLTPER
201 FORMAT(7X,5F6.0,2F6.2,F6.0,F6.2,F7.2)
C
C      BATTLE TERMINATION SECTOR
200 IF( RINF/RINFZ .LT. ENDFR .OR. BINF/BINFZ .LT. ENDFR) GO TO 211
      IF(RINF .LE. 1. .OR. BINF .LE. 1.) GO TO 211
      IF(BAMM .GT. BINF .AND. RAMM .GT. RINF) GO TO 1
211 IF(NXTPRT-PRTPER .EQ. TIME) RETURN
      JSWIT=1
      GO TO 210
212 WRITE(6,100) TIME,BINF,RINF,BAMM,RAMM

```

```

WRITE(6,201) MOE
RETURN
C
C
END
C
FUNCTION TVEFF(TIME,VZERO,VTC)
C   FUNCTION FOR TIME VARYING MULT. EFFECT ON INF FIRE EFF
TVEFF=1.-(1.-VZERO)*EXP(-TIME/VTC)
RETURN
END
C
FUNCTION FAAIM(RATIO,XMAXEF,CRITRT)
C   FUNCTION FOR MULT EFF ON AMMO CONST ON FIRE EFFICIENCY
XX=-RATIO/CRITRT
IF(XX .LT. -3.) GO TO 1
FAAIM=1.-(1.-XMAXEF)*EXP(XX)
RETURN
1 FAAIM=1.
RETURN
END
C
FUNCTION FSHOTT(RATIO,CRTRAT)
XX=-RATIO/CRTRAT
IF(XX .LT. -10) GO TO 1
FSHOTT=1.-EXP(XX)
RETURN
1 FSHOTT=1.
RETURN
END
C
FUNCTION STEP(S[H,STT)

```

```

STEP=0.
IF(STT .GE. 0.) STEP=STH
RETURN
END

```

C

```

SUBROUTINE PLOT0(I1,I2,I3,I4,I5,I6)
PLOT GRAPH OUTPUT ROUTINE

```

C

C

```

INTEGER LINE(101),LSP(5,10),NLS(5),LNS(14),LABEL(24)
INTEGER LETTER(15),MESAG(11),NAME(15)
REAL SCALE(15,2)
INTEGER ILIST(6),IFLAG(6),SINDEX(6)
INTEGER DOT,SPC,CCM,MIN,LEN,WID
REAL DATA(14)
COMMON / / NRUN,NSPOOL,LETTER,SCALE,MESAG,NAME
DATA DOT,SPC,CCM,MIN/1H.,1H ,1H.,1H-/

```

C

C

```

GET NVAR--NUMBER OF VARIABLES THIS PLOT
ILIST(1)=I1
ILIST(2)=I2
ILIST(3)=I3
ILIST(4)=I4
ILIST(5)=I5
ILIST(6)=I6

```

C

```

NVAR=0
DO 1 I=1,6
IF(ILIST(I) .EQ. 0) GO TO 1
NVAR=NVAR+1
ILIST(I)=ILIST(I)-100
1 CONTINUE
IF(NVAR .EQ. 0) RETURN

```

C

```
LCT=1
ZZLEN=48.
LEN=49
WID=12
REWIND 99
WRITE(6,2) NRUN,MESAG
2 FORMAT(*1*///20X,*RUN NUMBER *,I4/5X,11A6//)
```

C

C

C

```
SCALE HEADINGS
DO 13 J=1,24
13 LABEL(J)=6H
K=0
DO 10 I=1,NVAR
II=ILIST(I)
K=K+1
LABEL(K)=NAME(II)
K=K+1
LABEL(K)=1H=
K=K+1
LABEL(K)=LETTER(II)
IF(NVAR .EQ. 1) GO TO 10
K=K+1
LABEL(K)=COM
10 CONTINUE
WRITE(6,12) LABEL
12 FORMAT(3X,6(A6,3A1))
```

C

```
DO 20 I=1,6
20 IFLAG(I)=0
DO 21 I=1,NVAR
```

```

      II=ILIST(I)
      IF(IFLAG(I) .NE. 0) GO TO 21
C
      XLOW=SCALE(II,1)
      XHIGH=SCALE(II,2)
      XMID=(XHIGH+XLOW)/2.
      DO 22 J=2,6
22      SINDEK(J)=SPC
      SINDEK(1)=LETTER(II)
C
      IF(I .EQ. NVAR) GO TO 25
      KK=2
      IBOT=I+1
      DO 23 K=IBOT,NVAR
      IF(XLOW .NE. SCALE(ILIST(K),1)) GO TO 23
      IF(XHIGH .NE. SCALE(ILIST(K),2)) GO TO 23
      SINDEK(KK)=LETTER(ILIST(K))
      IFLAG(K)=1
      KK=KK+1
23      CONTINUE
C
25      WRITE(6,28) XLOW, XMID, XHIGH, SINDEK
28      FORMAT(2X,E3.2,16X,E3.2,16X,E3.2,1X,6A1)
C
21      CONTINUE
C
      DO 1210 NCOUNT=1,NSPOOL
C
      READ SPOOLED DATA FOR ONE PLOT PERIOD
      READ (99) TIME, DATA
C
C

```

```

C      BLANK OUT LINE IMAGE
DO 1030 I=1,LEN
LINE(I)=SPC
1030 CONTINUE
C
      NSP=0
C      NUMBER OF SUPERPOSITIONS THIS LINE
C      PLACE PLOT CHARACTERS FOR EACH OF NVAR VARIABLES
DO 1090 N=1,NVAR
TEMP=(DATA(ILIST(N))-SCALE(ILIST(N),1))/(SCALE(ILIST(N),2)-
1 SCALE(ILIST(N),1))
TEMP=TEMP*ZZLEN+1.5
I=TEMP
IF(I .GT. LEN) GO TO 1090
IF(I .LE. 0) GO TO 1090
IF(LINE(I) .EQ. SPC) GO TO 1080
C
C      HAVE A SUPERPOSITON
IF(NSP .EQ. 0) GO TO 1060
C
DO 1050 M=1,NSP
IF(LSP(M,1) .EQ. LINE(I)) GO TO 1070
1050 CONTINUE
C
1060 NSP=NSP+1
LSP(NSP,1)=LINE(I)
LSP(NSP,2)=LETTER(ILIST(N))
NLS(NSP)=2
GO TO 1090
C
C      MANY SUPERPOSITIONS IN THIS COLUMN
C

```

```

1070 J=NLS(M)+1
    LSP(M,J)=LETTER(ILIST(N))
    NLS(M)=J
    GO TO 1090
C
1080 LINE(I)=LETTER(ILIST(N))
1090 CONTINUE
C
C      END OF LOOP TO PLACE NVAR VARIABLES
C
C
C      K=1
C      NEXT ELEMENT OF LNS( ) TO BE FILLED
C      IF(NSP .EQ. 0) GO TO 1130
C
C      LOOP TO FILL IN LNS
C      DO 1120 L=1,NSP
C      IF(K .EQ. 1) GO TO 1100
C      LNS(K)=COM
C      K=K+1
1100 J=NLS(L)
C
C      DO 1110 I=1,J
C      LNS(K)=LSP(L,I)
1110 K=K+1
C
1120 CONTINUE
C
C      END OF LOOP TO FILL IN LNS
1130 DO 1140 I=K,14

```



```

      LNS(I)=SPC
1140 CONTINUE
C
C      EMBED DOTS EVERY WID SLOTS
      DO 1150 I=1,LEN,WID
      IF(LINE(I) .EQ. SPC) LINE(I)=DOT
1150 CONTINUE
C
      LCT=LCT-1
      IF(LCT .EQ. 0) GO TO 1180
      IF(NCOUNT .EQ. NSPOOL) GO TO 1180
C
C      DONT FILL IN DASHES
      WRITE(6,1170) (LINE(J),J=1,LEN),SPC,LNS
1170 FORMAT(7X,116A1)
      GO TO 1210
C
C      FILL IN DASHES
1160 DO 1190 I=3,99,2
      IF(LINE(I) .EQ. SPC) LINE(I)=MIN
1190 CONTINUE
      LCT=10
      WRITE(6,1200) TIME,(LINE(J),J=1,LEN),SPC,LNS
1200 FORMAT(F7.2,116A1)
C
1210 CONTINUE
      RETURN
      END

```

APPENDIX C

MODEL OUTCOMES FOR SELECTED ENGAGEMENTS

ORIGINAL PARAMETERS
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV

1	SOLUTION STEP SIZE	=	.250
2	PRINT PERIOD	=	2.000
3	PLOT PERIOD	=	1.000
4	VARIABLE STEP SIZE MAX %	=	0.000
5	END BATTLE FORCE RATIO	=	.020
6	INITIAL BLUE INFANTRY	=	75.000
7	INITIAL RED INFANTRY	=	50.000
8	GAMMA DENSITY CORRECTION	=	0.000
9	RED AMMO CRIT MIN INV	=	1.000
10	BLUE AMMO CRIT MIN INV	=	1.000
11	BASE EFF BLUE INF FIRE	=	.001
12	BASE EFF RED INF FIRE	=	.001
13	BASE EFF BLUE ARTY FIRE	=	.050
14	BASE EFF RED ARTY FIRE	=	.050
15	BASE RATE FIRE RED INF	=	40.000
16	BASE RATE FIRE BLUE INF	=	40.000
17	RED S+D PER RED CASUAL	=	0.000
18	BLUE S+D PER BLUE CASUAL	=	0.000
19	RED S+D DUE FORCE RATIO	=	0.000
20	BLUE S+D DUE FORCE RATIO	=	0.000
21	INT BLUE AMMO PER MAN	=	10000.000
22	INT RED AMMO PER MAN	=	10000.000
23	DELAY CALL BLUE ARTY	=	10.000
24	DELAY CALL RED ARTY	=	5.000
25	% RED SUPP OF BLUE INF	=	0.000
26	% BLUE SUPP OF RED INF	=	0.000
27	NUM BLUE ARTY TUBES	=	0.000
28	NUM RED ARTY TUBES	=	0.000
29	BLUE INF TV EFF AT ZERO	=	1.000
30	BLUE INF TV EFF TC	=	1.000
31	RED INF TV EFF AT ZERO	=	1.000
32	RED INF TV EFF TC	=	1.000
33	BLUE AMMO CONS FIRE EFF	=	1.000
34	RED AMMO CONS FIRE EFF	=	1.000
35	BLUE RORD FT AMMO/MAN	=	0.000
36	RED RORD FT AMMO/MAN	=	0.000
37	BLUE ORD UP TO AMMO/MAN	=	10000.000
38	RED ORD UP TO AMMO/MAN	=	10000.000
39	BLUE AMMO DELAY TIME	=	0.000
40	RED AMMO DELAY TIME	=	0.000

PERMANENT PARAMETER CHANGES

PARAMETER	6	INITIAL BLUE INFANTRY			
		NEW VALUE	50.000	REPLACES	75.000
PARAMETER	21	INT BLUE AMMO PER MAN			
		NEW VALUE	300.000	REPLACES	10000.000
PARAMETER	22	INT RED AMMO PER MAN			
		NEW VALUE	300.000	REPLACES	10000.000
PARAMETER	35	BLUE RORD FT AMMO/MAN			
		NEW VALUE	200.000	REPLACES	0.000
PARAMETER	36	RED RORD FT AMMO/MAN			
		NEW VALUE	200.000	REPLACES	0.000
PARAMETER	37	BLUE ORD UP TO AMMO/MAN			
		NEW VALUE	300.000	REPLACES	10000.000
PARAMETER	38	RED ORD UP TO AMMO/MAN			
		NEW VALUE	300.000	REPLACES	10000.000
PARAMETER	39	BLUE AMMO DELAY TIME			
		NEW VALUE	20.000	REPLACES	0.000
PARAMETER	40	RED AMMO DELAY TIME			
		NEW VALUE	4.000	REPLACES	0.000

PARAMETER 28 NUM RED ARTY TUBES

NEW VALUE

1.000

REPLACES

0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	42. 8.	46. 4.	17.	7.	10.	2.30	2.30	11307. 5.	.90	11138. .10
4.00	34. 16.	43. 7.	32.	13.	18.	2.35	2.35	8256. 9.	.79	7564. .21
6.00	29. 21.	41. 9.	42.	16.	24.	2.29	2.29	5754. 12.	.71	4296. .29
8.00	26. 24.	39. 11.	46.	23.	25.	2.11	2.11	3580. 13.	.67	14965. .33
10.00	23. 27.	37. 13.	54.	27.	28.	2.04	2.04	1744. 14.	.62	11947. .38
12.00	20. 30.	36. 14.	60.	29.	31.	2.07	2.07	510. 15.	.56	9057. .44
14.00	17. 33.	35. 15.	66.	30.	36.	2.19	2.19	76. 18.	.49	6249. .51
15.50	15.	35.						14.		4195.
	35.	15.	70.	30.	40.	2.32	2.32	20.	.43	.57

RUN NUMBER 1
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CONST

BINF=B, RINF=R, MOE10=0, BAMB=Y, RAMM=Z

0.		.25E+02		.50E+02 BR
-.30E+01		-.10E+01		.10E+01 0
0.		.75E+04		.15E+05 YZ
0.00.	- - - - -		0 - - - - -	B BRYZ
.	.	.	.0	Y B R . YZ
.	.	.	Y0 B	R . YZ
.	.	.	B 0	R .
.	.	Z Y	.	0 R .
.	.	Z Y	B	. 0R .
.	.	Z Y	B	. 0R .
.	Z	Y	B	. R 0 .
.	Z	Y	B	. R 0 .
.	Y	.	B	. R 0 .
.	Y	B	R	0 Z .
10.00.	- - Y - -	B	R Z	- 0 - -
.	Y	B	ZR	0 .
.	Y	B	R	0 .
.	Y	B	R	0 .
15.00Y	- - - - -	BZ	R	- 0 - -

NEW PARAMETER CHANGES

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 2.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	37. 13.	46. 4.	25.	7.	18.	3.59	3.59	11465. 9.	.80	11135. .20
4.00	27. 23.	44. 6.	46.	12.	33.	3.72	3.72	8853. 17.	.62	7535. .38
6.00	21. 29.	42. 8.	59.	16.	42.	3.64	3.64	6958. 21.	.49	4213. .51
8.00	18. 32.	40. 10.	65.	19.	45.	3.37	3.37	5405. 23.	.44	14765. .56
10.00	15. 35.	39. 11.	71.	22.	49.	3.26	3.26	4103. 25.	.37	11581. .63
12.00	11. 39.	38. 12.	77.	24.	53.	3.23	3.23	3052. 27.	.30	6496. .70
14.00	8. 42.	37. 13.	83.	26.	58.	3.26	3.26	2245. 29.	.23	5511. .77

16.00	6.	37.						1671.		2742.
	44.	13.	89.	27.	62.	3.33	3.33	31.	.15	.85
18.00	3.	36.						1307.		11637.
	47.	14.	94.	27.	67.	3.44	3.44	33.	.08	.92
19.50	1.	36.						1184.		9464.
	49.	14.	98.	28.	71.	3.56	3.56	35.	.02	.98

BINF=B, RINF=R, MOE10=0, BAMM=Y, RAMM=Z

90

PARAMETER 26 NUM RED ARTY TUBES
 NEW VALUE 3.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	33. 17.	47. 3.	33.	7.	26.	4.89	4.89	11616. 13.	.72	11132. .29
4.00	21. 29.	44. 6.	57.	11.	46.	5.09	5.09	9386. 23.	.48	7509. .52
6.00	15. 35.	43. 7.	71.	14.	57.	5.05	5.05	7973. 28.	.34	4140. .66
8.00	11. 39.	42. 8.	77.	16.	61.	4.77	4.77	6918. 30.	.27	14846. .73
10.00	3. 42.	41. 9.	84.	18.	66.	4.72	4.72	6121. 33.	.20	11523. .80
12.00	5. 45.	41. 9.	90.	19.	71.	4.80	4.80	5587. 36.	.12	8262. .88
14.00	2. 48.	40. 10.	97.	19.	77.	4.99	4.99	5312. 39.	.04	5080. .96
14.50	1. 49.	40. 10.	98.	19.	79.	5.05	5.05	5283. 39.	.02	4313. .98

RUN NUMBER 3
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CONST

BINF=B, RINF=R, MOE10=0, BAMB=Y, RAMM=Z

0.				.25E+02				.50E+02	BR							
-.30E+01				-.10E+01				.10E+01	0							
0.				.75E+04				.15E+05	YZ							
0.00.	-	-	-	-	-	-	-	0	-	-	-	-	B	BRYZ		
.0B	Y	R	.	YZ				
.	B	ZY	0		R	.					
.B	Z	Y	.	0	R	.					
.	B	Z	Y	.		0R	.					
.	B	Z	Y	.		0R	.					
.	ZB	.	Y	.		R	0	.				
.	Z	B	Y	.		R	0	.				
.	B	.	Y	.		R	0	Z				
.	B	.	Y	.		R	Z	0	.			
10.00.	-	-	-	B	-	-	-	Y	-	-	-	-	Z-R	-	0	.
.	B	.	.	Y	.	.	Z	.	R	.	0	.
.	B	.	.	Y	.	.	Z	.	R	.	0	.
.	B	.	.	Y	Z	.	.	.	R	.	0	.
14.00.	B	-	-	-	-	-	-	ZY	-	-	-	-	-R	-	-	0

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
NEW VALUE 1.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 1.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	42. 8.	42. 8.	17.	17.	0.	1.00	1.00	11304. 0.	1.00	11304. 0.00
4.00	35. 15.	35. 15.	31.	31.	0.	1.00	1.00	8226. 0.	1.00	8226. 0.00
6.00	30. 20.	29. 21.	40.	42.	-2.	.95	.95	5655. -1.	1.04	5666. -.04
8.00	28. 22.	24. 26.	44.	52.	-8.	.85	.85	3372. -4.	1.17	3555. -.17
10.00	26. 24.	20. 30.	47.	60.	-13.	.79	.79	1442. -6.	1.32	4891. -.32
12.00	25. 25.	18. 32.	50.	63.	-13.	.80	.80	319. -6.	1.34	3372. -.34

14.00	23.	18.						39.	1946.
	27.	32.	53.	64.	-10.	.84	.84	-5.	1.28
									-.28
14.50	23.	18.						22.	1610.
	27.	32.	54.	64.	-10.	.85	.85	-5.	1.27
									-.27

RUN NUMBER 4
LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CONST

BINF=B, RINF=R, MOE10=0, BAMB=Y, RAMB=Z

0.	.25E+02	.50E+02 BR
-.30E+01	-.10E+01	.10E+01 0
0.	.75E+04	.15E+05 YZ
0.00.	- - - - -	0 - - - - - B BRYZ
.	.	0 Y B . BR,YZ
.	.	0 B . BR,0YZ
.	.	0B . BR,YZ
.	. Y . B 0	. BR,YZ
.	. Y . B 0	. BR,YZ
.	. Y . B 0	. YZ
.	. YZ . R. B 0.	. YZ
.	. Y . R. B 0.	.
.	. Y Z . R . B 0 .	.
10.00.	- - Y - - - Z - R -	. B - - - 0 - - - - -
.	. Y . Z . R . B 0 .	.
.	. Y . Z . R . B 0 .	.
.	. Y . Z . R . B 0 .	.
14.00Y	- - Z - - - - -	- 0 - - - - -

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
NEW VALUE 1.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 2.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	37. 13.	42. 8.	25.	16.	9.	1.54	1.54	11462. 4.	.90	11301. .10
4.00	28. 22.	35. 15.	45.	29.	15.	1.51	1.51	8823. 8.	.79	8197. .21
6.00	22. 28.	30. 20.	56.	40.	16.	1.41	1.41	6862. 8.	.73	5577. .27
8.00	20. 30.	26. 24.	61.	49.	12.	1.24	1.24	5190. 6.	.77	3374. .23
10.00	18. 32.	22. 28.	64.	57.	8.	1.13	1.13	3662. 4.	.82	4748. .18
12.00	16. 34.	20. 30.	68.	60.	8.	1.13	1.13	2327. 4.	.81	3096. .19

14.00	15.	19.						1159.		1612.
	35.	31.	70.	62.	8.	1.13	1.13	4.	.78	.22
16.00	13.	13.						344.		6430.
	37.	32.	73.	64.	9.	1.15	1.15	5.	.74	.26
18.00	12.	18.						51.		4997.
	38.	32.	76.	64.	12.	1.18	1.18	6.	.67	.33
19.50	11.	16.						10.		3933.
	39.	32.	78.	65.	14.	1.21	1.21	7.	.61	.39

BINF=B, RINF=R, MOE10=0, BAMM=Y, RAMM=Z

98

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
 NEW VALUE 1.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
 NEW VALUE 3.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	34. 16.	42. 8.	33.	16.	17.	2.05	2.05	11613. 8.	.20	11298. .20
4.00	22. 28.	36. 14.	56.	28.	28.	1.97	1.97	9358. 14.	.61	8171. .39
6.00	16. 34.	31. 19.	69.	38.	31.	1.81	1.81	7883. 15.	.50	5499. .50
8.00	13. 37.	27. 23.	73.	46.	27.	1.59	1.59	6714. 14.	.50	3222. .50
10.00	11. 39.	23. 27.	77.	53.	24.	1.45	1.45	5714. 12.	.49	4646. .51
12.00	10. 40.	22. 28.	81.	56.	25.	1.45	1.45	4863. 13.	.43	2858. .57

14.00	8.	22.						4149.		3641.
	42.	28.	84.	57.	27.	1.47	1.47	13.	.38	.62
16.00	6.	21.						3563.		6644.
	44.	29.	87.	58.	29.	1.50	1.50	15.	.30	.70
18.00	5.	20.						3111.		4988.
	45.	30.	91.	59.	32.	1.53	1.53	16.	.23	.77
20.00	3.	20.						2791.		3373.
	47.	30.	94.	60.	34.	1.57	1.57	17.	.15	.85
22.00	2.	20.						2598.		1834.
	48.	30.	97.	60.	37.	1.61	1.61	18.	.08	.92
23.00	1.	20.						2546.		1151.
	49.	30.	98.	60.	38.	1.63	1.63	19.	.04	.96

RUN NUMBER 6
LANCHESTER INFANTRY MODEL WZARTY,RESUP,DELAY,TV COEFF,INV CONST

[illegible]

PARAMETER 27 NUM BLUE ARTY TUBES
 NEW VALUE 2.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
 NEW VALUE 1.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	42. 8.	37. 13.	16.	25.	-9.	.65	.65	11301. -4.	1.12	11462. -.12
4.00	35. 15.	28. 22.	29.	45.	-15.	.66	.66	8197. -8.	1.27	8823. -.27
6.00	31. 19.	20. 30.	38.	59.	-22.	.63	.63	5566. -11.	1.54	6879. -.54
8.00	30. 20.	14. 36.	40.	71.	-31.	.57	.57	3188. -15.	2.08	5474. -1.08
10.00	29. 21.	10. 40.	42.	80.	-38.	.53	.53	1217. -19.	2.90	4490. -1.90
12.00	28. 22.	9. 41.	44.	83.	-39.	.53	.53	226. -19.	3.23	3755. -2.23
14.00	27. 23.	8. 42.	45.	83.	-38.	.54	.54	25. -19.	3.23	3069. -2.23

RUN NUMBER 7
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CCNST

BINP=B, RINF=R, MOE10=0, BAMB=Y, RAMM=Z
 0. .25E+02 .50E+02 BR
 -.30E+01 -.10E+01 .10E+01 0
 0. .75E+04 .15E+05 YZ
 0.00. - - - - - 0 - - - - B BRYZ
 0. R B . RYZ
 0RZ B . RY
 RZ 0 .B . RY
 YRZ 0B .
 YR.Z 0B .
 YR Z . 0B .
 Y R Z . 0 B .
 Y . R Z 0 . B .
 Y R. Z 0 . B .
 10.00. - Y - - R .0Z - - - - B - - - - .
 . Y . 0D .Z . B .
 .Y . RU Z . B .
 Y . RD Z . B .
 14.00Y - - - RDZ . - - - - B - - - - .

NEW PARAMETER CHANGES

PARAMETER 27 NUM ELUE ARTY TUBES
NEW VALUE 2.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 2.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	38. 12.	38. 12.	25.	25.	0.	1.00	1.00	11459. 0.	1.00	11459. 0.00
4.00	29. 22.	28. 22.	43.	43.	0.	1.00	1.00	8796. 0.	1.00	8796. 0.00
6.00	23. 27.	21. 29.	54.	58.	-3.	.94	.94	6775. -2.	1.08	6793. -.08
8.00	21. 29.	16. 34.	57.	69.	-12.	.83	.83	4995. -6.	1.37	5295. -.37
10.00	20. 30.	11. 39.	59.	77.	-18.	.77	.77	3329. -9.	1.80	4193. -.80
12.00	20. 30.	9. 41.	61.	81.	-20.	.75	.75	1795. -10.	2.06	3363. -1.06

14.00	19. 31.	8. 42.	62.	83.	-21.	.75.	.75	600. -11.	2.27	2654. -1.27
16.00	18. 32.	8. 42.	64.	84.	-21.	.75	.75	96. -10.	2.33	2012. -1.33
17.75	18. 32.	8. 42.	65.	65.	-20.	.76	.76	14. -10.	2.29	1472. -1.29

RUN NUMBER 8
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CONST

BINF=B, RINF=R, MOE10=0, BAMB=Y, RAMM=Z
 0. .25E+02 .50E+02 BR
 -.30E+01 -.10E+01 .10E+01 0
 0. .75E+04 .15E+05 YZ
 0.00. - - - - - 0 - - - - - B BRYZ
 0 B . BRYZ
 BY . BR0,YZ
 BY 0 . BR,YZ
 BY 0 . BR,YZ
 BY 0 . BR,YZ
 R B . 0 . BYZ
 RY B . 0 . YZ
 RYZ B . 0 .
 R Z B . 0 . RY
 10.00. - - - - - R Z - - - - - B 0 - - - - - RY
 Y R Z B 0 .
 Y R Z B 0 .
 Y R Z B 0 .
 Y R Z B 0 .
 Y ZR B 0 .
 Y ZR B 0 .
 17.00Y - - Z R - - - - - B 0 - - - - -

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
 NEW VALUE 2.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
 NEW VALUE 3.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	34. 16.	38. 12.	33.	25.	8.	1.33	1.33	11610. 4.	.89	11456. .11
4.00	23. 27.	29. 21.	55.	43.	12.	1.29	1.29	9331. 6.	.78	8771. .22
6.00	17. 33.	22. 28.	67.	56.	11.	1.20	1.20	7801. 5.	.75	6717. .25
8.00	15. 35.	17. 33.	70.	66.	4.	1.05	1.05	6528. 2.	.89	5142. .11
10.00	14. 36.	13. 37.	72.	75.	-2.	.97	.97	5367. -1.	1.09	3944. -.09
12.00	13. 37.	11. 39.	74.	77.	-3.	.96	.96	4292. -2.	1.14	2958. -.14

14.00	12. 38.	10. 40.	76.	79.	-3.	.96	.96	3290. -2.	1.17	2120. -.17
16.00	11. 39.	9. 41.	77.	81.	-4.	.95	.95	2357. -2.	1.20	1338. -.20
18.00	11. 39.	9. 41.	79.	83.	-4.	.95	.95	1492. -2.	1.24	664. -.24
20.00	10. 40.	8. 42.	80.	84.	-4.	.95	.95	725. -2.	1.28	1072. -.28
22.00	9. 41.	7. 43.	81.	86.	-4.	.95	.95	200. -2.	1.30	516. -.30
24.00	9. 41.	7. 43.	82.	86.	-4.	.96	.96	28. -2.	1.25	2504. -.25
25.25	8. 42.	7. 43.	83.	86.	-3.	.97	.97	7. -1.	1.21	2152. -.21

RUN NUMBER 9
 LANCHESTER INFANTRY MODEL W/ARTY, RESUP, DELAY, TV COEFF, INV CONST

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    BINF=B, RINF=R, MOE10=0, BAMB=Y, RAMB=Z
0.      .25E+02      .50E+02 BR
-.30E+01      -.10E+01      .10E+01 0
0.      .75E+04      .15E+05 YZ
0.00.  - - - - - 0 - - - - - B BRYZ
      .      .      .      .0 B R      . RYZ
      .      .      .      B R0      . OYZ
      .      .      .      . B RY      . 0      . RZ
      .      .      .      B R Y      . 0      . RZ
      .      .      .      B RZ Y      . 0      .
      .      .      .      B R Y      . 0      . RZ
      .      .      .      B R Y      . 0      . RZ
      .      .      .      B R Y      . 0      . RZ
      .      .      .      B R Y      . 0      . RZ
      .      .      .      B R Y      . 0      . BRZ
10.00.  - - - - - RB - Y - - - - - 0 - - - - - BZ
      .      .      .      .0      .      . RZ
      .      .      .      ZRB Y      .      .
      .      .      .      Z R B      .      . BY
      .      .      .      Z RYB      .      .
      .      .      .      Z R B      .      . RY
      .      .      .      Z Y R B      .      .
      .      .      .      Z Y R B      .      .
      .      .      .      YZ R B      .      .
20.00.  YZ - - - - - RB - - - - - 0 - - - - -
      .YZ R B      .      .0      .
      .YZ R B      .      .0      .
      Y R B      .      .0      . BZ
      Y R B      .      .0      . BZ
25.00Y - - - - - RB - - - - - 0 - - - - - RZ
  
```

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
NEW VALUE 3.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 1.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	42. 8.	34. 16.	16.	33.	-17.	.49	.49	11298. -8.	1.25	11613. -.25
4.00	36. 14.	22. 23.	28.	56.	-28.	.51	.51	8171. -14.	1.63	9353. -.63
6.00	32. 18.	14. 36.	36.	72.	-37.	.49	.49	5487. -16.	2.32	7901. -1.32
8.00	31. 19.	8. 42.	37.	84.	-46.	.45	.45	3037. -23.	3.88	7008. -2.88

10.00	31.	4.						1059.	6511.
	19.	46.	38.	92.	-53.	.42	.42	-27. 7.30	-6.30
12.00	31.	3.						175.	6223.
	19.	47.	39.	94.	-55.	.42	.42	-27. 9.60	-8.60
13.75	30.	3.						25.	6007.
	20.	47.	39.	94.	-55.	.42	.42	-27. 10.01	-9.01

RUN NUMBER 10
 LANCHESTER INFANTRY MODEL W/ARTY,RESUP,DELAY,TV COEFF,INV CONST

BINF=B, RINF=R, NOE10=0, BAMB=Y, RAMM=Z

0.			.25E+02			.50E+02 BR
-.30E+01			-.10E+01			.10E+01 0
0.			.75E+04			.15E+05 YZ
0.00.	-	-	-	-	0	B BRYZ
.	0. R Y B	. YZ
.	.	.	.	R0 YZ B	.	.
.	.	.	R	0 Z .B	.	0Y
.	.	R	Y 0 Z B	.	.	.
.	.	R	Y 0 Z B	.	.	.
.	R	Y 0	Z B	.	.	.
.	R	Y	Z B	.	.	.
.0	R Y	.	Z B	.	.	.
.	R	.	Z B	.	.	RY
10.00.	-YR	-	-Z-	-	B-	.
.	YR	.	Z	.	B	.
.	Y R	.	Z	.	B	.
13.00Y	-R-	-	-Z-	-	B-	.

NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
NEW VALUE 3.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 2.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	38. 12.	34. 16.	25.	33.	-8.	.75	.75	11456. -4.	1.12	11610. -.12
4.00	29. 21.	23. 27.	43.	55.	-12.	.77	.77	8771. -6.	1.28	9331. -.28
6.00	24. 26.	15. 35.	52.	70.	-18.	.74	.74	6699. -9.	1.62	7820. -.62
8.00	23. 27.	9. 41.	54.	82.	-27.	.66	.66	4831. -14.	2.48	6844. -1.48
10.00	22. 28.	5. 45.	55.	90.	-34.	.62	.62	3046. -17.	4.26	6253. -3.26
12.00	22. 28.	3. 47.	56.	93.	-37.	.60	.60	1423. -19.	6.42	5907. -5.42

14.00	22.	2.						352.	5676.
	28.	48.	57.	95.	-39.	.59	.59	-19. 9.25	-8.25
16.00	22.	2.						45.	5502.
	28.	48.	57.	96.	-39.	.59	.53	-20. 10.55	-9.55
16.75	21.	2.						19.	5441.
	29.	48.	57.	96.	-39.	.59	.59	-19. 10.66	-9.66

LANCHESTER INFANTRY MODEL W/ARTY,RESUP,DELAY,TX COEFF,INV CONST

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BINF=B, RINF=R, MOE10=0, BMM=Y, RAMM=Z
0. .25E+02 .50E+02 BR
-.30E+01 -.10E+01 .10E+01 0
0. .75E+04 .15E+05 YZ
0.00. - - - - - 0 - - - - B BRYZ
. . . . . 0. R B . BYZ
. . . . . R 0BY . YZ
. . . . . R BZ0 . BY
. . . . . R B Z 0 . BY
. . . . . R BY Z 0 .
. . . . . R Y B Z 0 .
. . . . . R Y BZ0 .
. . . . . R Y 0 B . BZ
. . . . . R 0 Y ZB .
10.00. - - - - - ZB - - - - -
. R Y . Z B .
. R Y . Z B .
. R . Z B . RY
. YR . Z B .
. YR . Z B .
16.00BY R - - - - - Z -B - - - - -

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NEW PARAMETER CHANGES

PARAMETER 27 NUM BLUE ARTY TUBES
NEW VALUE 3.000 REPLACES 0.000

PARAMETER 28 NUM RED ARTY TUBES
NEW VALUE 3.000 REPLACES 0.000

TIME	BINF MOE1	RINF MOE2	MOE3	MOE4	MOE5	MOE6	MOE7	BAMM MOE8	MOE9	RAMM MOE10
0.00	50. 0.	50. 0.	0.	0.	0.	99.99	99.99	15000. 0.	1.00	15000. 0.00
2.00	34. 16.	34. 16.	32.	32.	0.	1.00	1.00	11607. 0.	1.00	11607. 0.00
4.00	23. 27.	23. 27.	54.	54.	0.	1.00	1.00	9307. 0.	1.00	9307. 0.00
6.00	17. 33.	16. 34.	65.	69.	-4.	.94	.94	7729. -2.	1.13	7749. -.13
8.00	16. 34.	10. 40.	67.	80.	-12.	.84	.84	6370. -6.	1.60	6702. -.60
10.00	16. 34.	6. 44.	60.	87.	-19.	.79	.79	5081. -9.	2.46	6025. -1.46
12.00	15. 35.	5. 45.	69.	90.	-21.	.77	.77	3838. -10.	3.10	5572. -2.10

14.00	15. 35.	4. 46.	70.	93.	-22.	.76	.76	2633. -11.	4.01	5220. -3.61
16.00	15. 35.	3. 47.	71.	95.	-24.	.74	.74	1484. -12.	5.69	4962. -4.69
18.00	15. 35.	2. 48.	71.	97.	-26.	.73	.73	535. -13.	8.90	4791. -7.90
20.00	14. 36.	1. 49.	71.	98.	-26.	.73	.73	92. -13.	12.13	4680. -11.13
21.75	14. 36.	1. 49.	71.	98.	-26.	.73	.73	13. -13.	12.92	4600. -11.92

LANCHESTER INFANTRY MODEL W/ARTY,RESUP,DELAY,TV COEFF,INV CONST

[illegible]

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